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A LABORATORY MANUAL

OF

PHYSICS.

BY

C. CANBY BALDERSTON,

ASSOCIATE EDITOR OF SHARPLESS AND PHILIPS'S NATURAL PHILOSOPHY.

ILLUSTRATED.

PHILADELPHIA:
J. B. LIPPINCOTT COMPANY.

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PREFACE.

THIS hand-book, as its title indicates, is designed as a guide to the student in actual Laboratory work in Physics. While directions for the experiments are not given with the minuteness of detail which characterizes many works of similar grade, it is believed they are sufficiently explicit for the average student to follow successfully with simple suggestions from the teacher, or even without such suggestions when they are not to be had.

If the student has not already pursued the subject of Physics in a text-book, he is recommended to take a text in connection with the course here given, as references are continually made which assume that he is acquainted with the ordinary facts, terms, and laws of Physics. This work is, in fact, so nearly supplementary to Sharpless and Philips's "Natural Philosophy," that a few subjects fully treated in the experiments of that book, are passed over here with simple reference to the fact.

The experiments here indicated cover all the laboratory work in Physics recommended by the "Committee of Ten" of the American Educational Association, except a very few of the more difficult and uncertain determinations. The course is, however, much fuller than the bare recommendations of that Committee suggest. It is intended to be accomplished by three hours' work per week for the school year. If, however, the student works by himself, or must hunt up or construct the apparatus for each hour's work during the hour, it will be found that four or even five hours per week may be spent upon it.

The time devoted to manual training in many of our best schools enables students to construct the more simple pieces of apparatus called for, and, with the facilities offered in a few of these schools, most of the articles may be constructed. But this is a work of time, and most schools must supply themselves from regular manufacturers of physical apparatus. It has been the author's aim to appeal to the student's every-day experience by asking him to apply the results of his experiments to the work of the world going on around him.

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MEASURING.

Experiment 1.—With a metre-stick graduated to millimetres, a yard-stick graduated to eighths of an inch, or a two-foot rule graduated to sixteenths of an inch, measure several articles within reach,—*e.g.*, the length and breadth of your note-book, the length, breadth, and height of the table on which you work and of your work-room. If you have both a metre-stick and a two-foot rule, measure each one by the other. In all the above cases read the measure as closely as possible, dividing the distance between the graduations by the eye, and immediately record the result without any attempt to remember it. When through with all the articles at command, go over the list again, record, and compare with the first record.

Similarly *weigh* several convenient articles with a spring balance, recording, reweighing, and comparing records.

Experiment 2.—**Averaging.**—Measure with all possible precision the width of a rough board in several places. Add the measurements, divide the sum by the number of measurements, and obtain the average width. If the board tapers perceptibly and regularly, the measurements must be taken at uniform distances apart. In a case like the last, compare the average width thus obtained with the width found in the middle.

Experiment 3.—**To Determine Small Quantities.**—Measure as accurately as possible the thickness of a book or tablet, and divide by the number of sheets for the thickness of one sheet.

Similarly, obtain the average distance between the lines on a sheet of ruled paper. Measure the distance between the lines directly before obtaining the average distance.

Experiment 4.—**Area and Volume.**—Measure accurately, and from your measurements obtain the average length and width of a table-top, and calculate its area.

Take the average dimensions of a brick and calculate its volume. Take the average dimensions of your work-room and calculate its volume.

Experiment 5.—**Mass by Comparison.**—Carefully measure a yard or a metre of heavy iron wire, say No. 6, 8, or 10, Brown and Sharpe's gauge, and accurately weigh the measured piece. Weigh a coil of fine wire of the same material. From Table No. 5, Appendix, and the weight of the yard of coarse wire calculate the length of the coil of fine wire.

To illustrate: Suppose we weigh No. 6 iron wire, and wish to determine the length of a coil of No. 26 iron wire. Suppose the No. 6 wire is found to weigh 480 grains per foot. Turning to the table, we find No. 6 wire to be .162 inch in diameter and No. 26 to be .01594 inch. The areas in cross-section are as the squares of the diameters, or in this case as $.162^2$ to $.01594^2$, which worked out shows that No. 6 wire is 103.2 times as heavy per foot as No. 26 wire, from which, and the weight of a given measure of the No. 6 wire, we readily compute the length of any weight of the fine wire.

METRIC WEIGHTS AND MEASURES.

In this exercise use the metric measures exclusively.

Experiment 6.—**Linear Measures.**—Measure the length of the table accurately with the metre-stick, reading to nearest millimetre.

Have an assistant mark your height accurately on the wall, and measure from the floor in metres and millimetres.

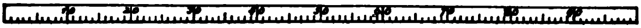

FIG. 1.

NOTE.—A metre-rod is graduated to millimetres, but the *numbers* on it indicate centimetres. The divisions of the metre are *read* in millimetres. Thus, 2 metres, 3 decimetres, 4 centimetres, and 6 millimetres is written 2.346 m., and is read 2 metres and 346 millimetres.

Experiment 7.—**To Determine Volumes.**—Measure the dimensions of a regular solid—*e.g.*, a brick—in millimetres, and calculate its volume in cubic millimetres.

Similarly determine the volume of your room in cubic metres.

Experiment 8.—**Relation of Volumes to Linear Measure.**—Measure very accurately the inside diameter and depth of a cylindrical litre measure, and calculate its capacity in cubic millimetres. Reduce to cubic centimetres and cubic decimetres (see Table 1, Appendix). Repeat this several times and take the average of all the results. Use an inside caliper for obtaining the diameter. How many cubic decimetres should be found in a litre?

Experiment 9.—**Derivation of Weight Units.**—Carefully balance on a scale-pan a vessel graduated in cubic centimetres. A hundred cubic centimetre flask, with mark in the neck, is the best. Put in a definite number of cubic centimetres of water and weigh again, using gram weights. The water should be clear, pure ice-water. Repeat this several times, wiping the vessel dry on the outside each time. One balancing of the vessel must do unless you can get at the inside and wipe it each time. Find the average weight of a definite number of cubic centimetres of water in grams, and from this determine the weight of one cubic centimetre of water.

On the larger scale-pan balance a litre flask, and then weigh it full of water. Take the average of several trials. Compare your results with Table 2, Appendix.

Experiment 10.—**To Measure a Vessel by the Weight of Water it will Contain.**—Balance any empty bottle on the scale-pan. Fill to the neck with water and weigh again in grams. From the weight of the water calculate the capacity of the bottle in cubic centimetres. A litre flask should contain how many times this bottle full? Try it.

STRETCHING AND ELASTICITY.

Experiment 11.—**To Determine the Relation of Strain to Stress.**—Take a piece of india-rubber cord (a rubber band, cut) and tie a loop in each end with waxed thread. Lay it very carefully, straight but not stretched, on the edge of a metre-stick, and tie two pieces of waxed thread around it a definite distance apart, say 10 centimetres. Now hook one loop over a nail driven in the table-top and the other to the light spring balance. Pull gently along the edge of the metre-stick till the length has *increased* say 5 centimetres. Note the reading of the balance. Release, see that the rubber resumes its original length, and if it does, pull again till the increase in length of the rubber is just twice, three times, etc., as great as the increase in the first case. How do the comparative spring-balance readings compare with the regular increase in length of the rubber?

Vary this experiment by hanging the rubber vertically from a support and attaching known weights to the lower end. If a scale-pan is used to hold the weights, it must be weighed and its weight counted as a weight. For this experiment *any* weights will do. For instance, suppose an extemporized scale-pan is found to be balanced on the balance by 10 nails from a certain keg. Hook the scale-pan to the lower rubber loop, and note the increase in length of the rubber (the part between the thread markers). Now put 10, 20, 30, etc., nails from the same keg into the scale-pan, and see how nearly the stretching is 2, 3, 4, etc., times as much as that produced by the empty pan.

Write a law stating the relation between stretching force and amount of stretching. Do not include any results found after the rubber is *permanently* stretched.

Experiment 12.—**Relation of Strain to Stress in Metals.**—Cut a piece of very fine spring brass or steel wire (No. 30 is not too fine) rather more than 2 metres in length. Securely loop a shoe-eyelet, or, better, a small sailmaker's eyelet, into each end of the wire. Tie waxed threads exactly 2 metres apart on the wire as markers. Now hook one eyelet on a nail in the table, and with the heavy spring balance hooked into the other eyelet and laid back down on the table, pull carefully say 2 pounds, and note the increase in length of the wire between the markers. Double the stress, and note the increase in length again. Repeat this several times, trebling, quadrupling, etc., the stress, and noting the increase in length. Note carefully the amount of stress when permanent elongation takes place.

Do you find that the stretching or *strain* of the wire is directly proportional to the stress? Write your conclusion and compare with that reached in case of the rubber.

Experiment 13.—**To Test the Spring Balance.**—To reach a conclusion in regard to the effect of regularly increasing stress or tension on a *spiral spring*, carefully measure the graduation of a spring balance, and with a number of uniform weights (nails will answer) test the accuracy of the balance. For instance, if 10 nails are required to move the index over *one* division of the scale, find how many are required to move it over the next, how many over the next, and so on.

Experiment 14.—**To Determine the Relation between Flexure and the Force which Produces It.**—Procure a stick of very good white pine, spruce, or cedar-wood, 1 metre by 3 centimetres by 1 centimetre, or 1 yard by 1 inch by ⅜ inch. Support it on two triangular supports, as shown, and place a weight of say ¼ pound, or 100 grams, in the middle. With a very light wire balanced on another support to serve as an index, a pin driven into the side of the stick to move this index and any scale of equal parts, read the amount of deflection of the bar. Double the weight and again read the deflection. Remove all the weight each time, and be sure the index comes to zero, or practically so, before applying the next heavier weight. Record the results, and when done note the relation between the amount of weight and amount of flexure. If this relation shows irregularity, there has probably been some mistake in the work. Do it over again. See that neither the long stick *a*, the end supports, the index *i*, nor any other part, is misplaced in the slightest by jostling.

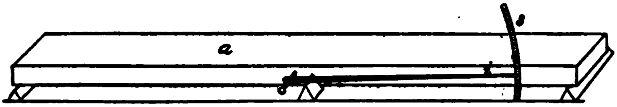

FIG. 2.

After getting a constant ratio between the bending force and the amount of bending, calculate the *exact amount* of bending in this stick for a given weight, say 100 grams. Of course, this is obtained from the motion of the index *i* on the scale *s*, taking exact account of the lengths of the two arms of the index *i*. Record this result, giving the exact length (between supports), the width, and thickness of the stick.

Experiment 15.—**Relation between Flexibility and Length.**—Repeat the above with a stick of the same material, same width and thickness, but *half* as long between supports. How much does the short stick bend with each load, as compared with the bending of the long stick under the same load? If you do not see a definite relation, try another short stick, or cut the long one in half. Satisfy yourself that you have found the relation between the flexibility and the *length* of the stick, and see how nearly it fulfils the law that "The rigidity of a beam of given thickness varies inversely as the cube of its length."

Experiment 16.—**Relation of Rigidity to Thickness of Beams.**—Take a stick of the same material as your first, with the length and breadth the same, but just *twice as thick.* The exact ratio of thickness here is essential. They must be machine made and accurate. Apply the weights, and record the results. If the bending is slight and difficult to read, use only the heaviest weight of those used on the thin rod, and record result. You were satisfied when working with the thin stick that the amount of flexure is exactly proportional to the weight used, or that "*strain is proportional to stress.*" Take now a weight eight times as heavy as the one last used and place it on the thick stick. How much is the actual flexure? It is difficult without *very exact* apparatus to make more than this first step in the proving of the very important law that "The rigidity of a stick varies as the third power of its thickness." You should here prove that 200 grams produce 8 times as much bending in a stick 1 centimetre thick as in a stick otherwise the same but 2 centimetres thick; also that 1600 grams on the stick 2 centimetres thick produce no more flexure than 200 grams on the stick 1 centimetre thick. You have simply to go further with the thickness of the stick and the amount of weight used to prove the general law.

We will not ask you to *prove* that the stiffness of a stick increases directly as its width. It is self-evident that two equal and similar planks side by side will support twice as much weight as one.

See that you have recorded:

(1) The ratio between flexure and weight applied.
(2) The ratio between flexure and length of beam.
(3) The ratio between flexure and width of beam.
(4) The ratio between flexure and thickness of beam.

Experiment 17.—**Comparative Strength of Beam on Flat Side and on Edge.**—For this experiment let the proportional dimensions of the stick be different, but the apparatus otherwise the same. For white pine wood, 60 centimetres by 1 centimetre by 2 centimetres will be found to work well. With the supports at the ends, try this stick on its side, then on its edge, and compare results. How do they agree with the results that you find from calculation, using the laws (3) and (4) above recorded? You are now ready to do some interesting figuring on the load required to bend given joists or girders to any extent.

Experiment 18.—**Breaking-Strength of Wires.**—Take up again the spring balance and wire of Experiment 12, only shorten the wire to 50 centimetres or less, and dispense with the markers. See that the nail holding one end is firm, and, taking a strong hold of the balance, pull and read carefully until the wire *breaks*. Two persons are better at this experiment,—one to read as the other pulls. He must hold his eyes so high that they will be out of reach of the recoiling ends of the broken wire. After breaking one piece, try one or two more of the same and average the results. Then try the same size and a different material. In this way determine the breaking-strength of iron, steel, brass, copper, using wires from 27 to 30 gauge, and compare the results.

From the diameters of wires, given in Table 5 of the Appendix, and the results of the experiments just performed calculate the breaking-strength of iron and steel rods per *square inch*. Your results may be higher than the figures given in engineers' tables, as *wires* (drawn) are stronger than *rods* (forged).

FORCE AND MOTION.

Experiment 19.—**Motion Proportional to Moving Force.**—Suspend two equal weights—say ½ pound each—on the ends of a string 3 or 4 yards long, which passes over a very easy-running pulley, as shown in Fig. 3. If there is any appreciable friction in the pulley after it has been thoroughly cleaned and oiled, add a little to the weight *a* till it will *nearly* fall, raising *b*. When satisfied that *b* is about ready to be moved by *a*, hang an additional weight *w*—say 1 ounce—on the string with *a*. Raise *a* or pull down *b* till the string is nearly all on *b*'s side, and hold the string between the thumb and finger to retain it till ready. Arrange a shelf, as shown, or get an assistant to hold it, to catch *a*, giving it say 2 or 3 feet to fall. A watch or clock ticking loudly enough to be heard is essential. When all is ready, let go the string just on any tick of the clock, and *count* while *a* descends and strikes on the board. The board may be moved up or down slightly till, after repeated trials, starting *a* from a given mark, it strikes the board just *on* a certain tick of the clock,—say the 4th, 6th, or 8th after starting. Now double the weight *w*, and see how far *a* descends during the same number of ticks, making several trials to be sure. From this you not only have an illustration of the fact that an unbalanced force produces motion, but that the amount of motion is directly proportional to the force used.

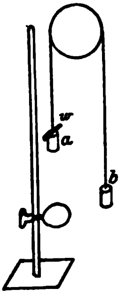

FIG. 3.

How nearly have you here proved Newton's second law of motion? Suppose *a* and *b* to have been 8 ounces each and *w* 1 ounce, how much is the moving force and what weight does it move?

Experiment 20.—**Momentum.**—Use for this experiment the two cars which will be needed for experiments with the Inclined Plane. See that the cars are oiled to run easily, and that the table-top or floor is smooth, hard, and level. Carefully weigh the cars, and load one of them so that it shall weigh 2, 3, or 4 times as much as the empty car.

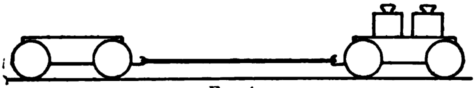

FIG. 4.

Place the cars face to face, with a light rubber band over the hooks. Lay the metre-stick on the table parallel with the cars, and pull the cars apart, stretching the rubber. Let both cars go at the same instant, and read on the metre-stick how far each moves by the force of the contracting rubber. Repeat several times, and take the average. How do the distances moved by the cars respectively compare with their weights? Multiply the weight of each car by the distance it moves. How do the results compare? At which end of the stretched rubber is the greater moving force exerted? According to the second law of motion, which car has the *greater amount of motion?* What is your understanding of the "momentum" of a moving body? Which car has the greater momentum?

Vary this experiment by making the cars of equal weight. Instead of the cars on a table, more accurate results may be obtained with a weak rubber band by connecting with it two weighed blocks floated on a tub of water.

Over a groove between two floor-boards place a glass marble, and roll against it carefully and rather smartly another marble of the same size, then one of half the size, then one of twice the size, and note carefully the effect each time on the marble struck.

Experiment 21.—**Composition of Forces.**—Place two pulleys against a vertical board, and see that they are free to turn. Arrange weights on a string, as shown in the figure. The weight *w* must not be greater than *a* and *b* combined. Tie the knot at *c* so that it will not slip. Let the weights balance themselves as they will; then hang a plumb-line above *f* so that it passes through the knot *c* and follows the line *cw*. Next measure off on the strings *cd*, *ce*, distances proportional to the weights *b* and *a*,—that is, if *b* is 8 ounces and *a* 6 ounces, make *cd* 8 inches, 8 centimetres, or 8 of some scale, and make *ce* 6 of the same scale. Through the points *d* and *e* draw lines parallel to *ce* and *cd*, meeting in *f*, and measure *fc*. How does its length compare with the weight of *w*? What line in the figure represents the *resultant* of *cd* and *ce*? What is the *equilibrant* of those forces? In what direction, as shown by the plumb-line, does the equilibrant act? What causes the equilibrant force, and in what direction does it act?

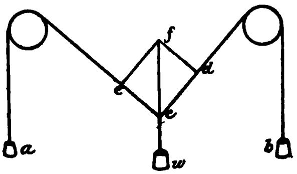

FIG. 5.

Experiment 22.—**Resolution of Forces.**—To resolve a force into two components, reverse the measuring of the preceding experiment. For instance, if *w* weighs 5 ounces and is pulling directly downward, hang a plumb-line through *c* and measure *up* 5 inches on *cf* for the single equilibrant to balance *w*. As, however, *w* is balanced jointly by *b* acting in the direction *cd* and *a* acting in the direction *ce*, draw through the point *f* two lines parallel respectively to *ce* and *cd*, and measure the distances *cd* and *ce* for the ratio of the two components of the force balancing *w*.

Experiment 23.—**To Determine a Unit of Work.**—Use again the pulley of Experiment 19. Hang it to a support at least a foot above the table. Hang a pound weight on each end, and with the pulley well oiled give the weight *A* a start downward, so that it will lift the weight *B* a foot before coming to rest. What force was necessary to raise *B* one foot, not counting friction, and how far (through what distance) did it move? Considering the *work* required to raise 1 pound 1 foot as a definite quantity, 1 *foot pound*, how do you show by figures that *A* does that work?

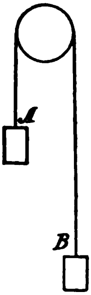

FIG. 6.

Experiment 24.—**Relation between Force and Distance it Acts.**—If one of the cars which you have used (Experiment 20) weighs less than a pound, weight it till it comes to just 1 pound weight. Attach a string to it, and pass it over a pulley which is set in the end of a very smooth hard board 4 or 5 feet long. With the board level, hang just enough weight at *P* to keep the 1-pound car moving. This is to balance the friction of the car. Now raise the end of the board and hang a 4-ounce weight at *P*, blocking up the board in such position that the 4-ounce weight and the weights previously added to balance friction shall just keep the car moving. When this point is reached, pull the car to the bottom of the plane, and by actual measure determine how far *P* falls to raise *W* 1 foot high. Put this apparatus on a high shelf, and give *P* room enough in its fall to raise *W* the whole foot, if you possibly can. If you cannot do this, measure when *W* has been raised 6 inches, and multiply by 2.

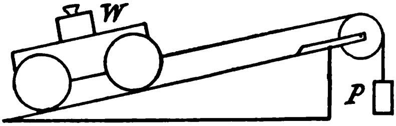

FIG. 7.

From these two experiments have you proved that a foot-pound of work is accomplished by a force of a pound acting through a foot? How do you account for the foot-pound of work done by the 4-ounce weight? State the general law showing relation between weight and distance it *rises*, and force and the distance it *moves*.

Experiment 25.—**Definite Energy of a Moving Mass.**—Place five or six good glass marbles in a straight row over the crack between two floor-boards. See that they are in absolute contact. Take another marble of the same size and roll it with a velocity of say 2 feet per second along the crack fair against the end of the row. Try it several times, and estimate as closely as you can the velocity with which the marble leaves the other end of the row.

Experiment 26.—**Kinetic and Potential Energy.**—Hang a weight of say 4 ounces by a string 15 inches long, suspended on a nail driven into the wall or, better, into the frame at the top of the black-board. Pull the weight back a considerable distance, and mark its height by a pencil or chalk. Let it swing freely, and catch it at the farthest point it reaches on the farther side. Mark this point. Try it several times, letting it go first from one side and then from the other, and be sure the marks are in the right places. Now measure the distances of these two marks above the ledge of the black-board, or draw a line between them, and ascertain its level.

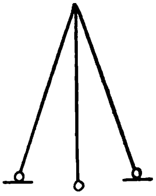

FIG. 8.

Formulate the results of the last two experiments. How do you express the moving, or kinetic, energy of the marble, and what did it accomplish? How do you express the potential energy of the weight when held back ready to swing? How have you proved that the kinetic energy of the weight at the lowest point of the swing was equal to its potential energy at the highest point? Vary the last experiment by hanging *two* equal elastic balls on strings of exactly the same length from the same pin. (Ivory balls are the best for this.) Carefully draw back one of these balls, and if the impact is fair, catch the second one at the end of its swing, using the same marks you used before. Again answer in regard to the relation between the potential and the kinetic energy at different phases of the swing.

Experiment 27.—**To Find the Centre of Gravity.**—Find and mark the centre of gravity in a piece of board by repeated trials at balancing it on the end of a penholder. Now bore a small hole in any other part of the board, and hang the board up by a wire nail through the hole. The board must turn very freely on the nail. Hang a plumb-line on the same nail, and observe how nearly it passes the point previously marked as the centre of gravity of the board. Try the board from several points, hanging the plumb-line on each time. When a body is suspended, in what direction is the centre of gravity from the point of support? How could you by this means determine the centre of gravity of a body?

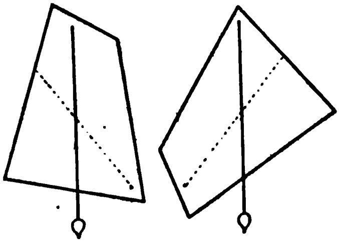

FIG. 9.

Experiment 28.—**Stability.**—Scotch a brick on a board. Carefully raise one end of the board till the brick falls over. Prop the board in this position, and leave it so while you work out the height to which one end of the board *should* be raised to throw the brick over in the position selected. Now measure the height of the board, the length of its base, etc., and see how near right you are. Try it in several positions. Where must the centre of gravity be with reference to the lower edge of the brick when the brick falls over? Draw a diagram showing how steep a plane a *cube* will rest on. Draw a diagram showing why a ball will not rest on an incline.

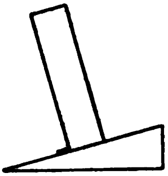

FIG. 10.

What effect has the *height* of a body upon its *stability?* Show by diagram that a common brick 2 × 4 × 8 inches is just twice as stable on its long edge as on its end,—that is, with the *length* of the supporting edge in each case *across* the slope of the plane. Try the above by experiment. What part of the brick must be cut by the line of direction at the moment when the brick falls over?

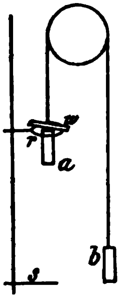

FIG. 11.

Experiment 29.—**Acceleration of a Mass acted on by a Constant Force.**—Use again the apparatus of Experiment 19. See that the pulley runs with the least possible amount of friction. With *a* and *b* balanced, *a* slightly heavier, if necessary, to overcome friction, place a weight *w*, in the shape of a rod, on the weight *a*, as shown. Have a clock ticking near. Lift *a* with its additional weight to the top, and let go just at a tick of the clock. Adjust the ring *r* at such height that the weight *w* shall strike it and lodge on it just at the first or second or third tick of the clock, depending on how rapidly the clock ticks, from a half second to a whole second all told. Say *w* strikes *r* just at the first tick after starting. Adjust the shelf *s* so that *a* will strike it just at the second tick of the clock. Carefully measure the distance from *r* to *s*. If the clock ticks seconds, this distance will give the velocity which the force represented by the weight of the rod *w* gives to the mass represented by the combined weights of *a*, *b*, and *w* in one second. If the clock does not tick seconds, or if it ticks very rapidly, and you count 2 or 3 ticks before catching *w* on the ring, and the same number of ticks additional before catching *a* on the shelf, then the distance from *r* to *s* represents the velocity acquired by the mass in a *unit of time*, no matter what that unit is.

Having by repeated trials satisfied yourself that you have the distance from *r* to *s* with tolerable accuracy, move *r* and *s* both down to such position that *r* shall catch *w* in just *two* of your adopted units of time, and *s* shall stop *a* in just *one* additional unit of time. Supposing again, for illustration, the unit of time to have been 1 second, *w* acted on the mass for 2 seconds before being stopped by *r*, therefore the distance from *r* to *s* is the velocity per second, which the same force, acting constantly for 2 seconds, gives to the same mass. After trying till you are again satisfied with your measurements, compare the velocity which the mass acquires in 2 seconds with the velocity which it acquires in 1 second. If your pulley is high enough, try this for the velocity gained in 3 seconds.

If your pulley was quite free from friction, you are now ready to state an important law, beginning with "the velocity of a given mass acted on by a constant force. . . ." Write it out in full from your determination. What is the force in the above case? If we call it the force of gravity, why not let gravity act on the weight *w* directly, free to fall, without holding it back by the weights *a* and *b*. If, as our works on mechanics tell us, gravity gives a body free to fall a velocity of 32.2 feet in 1 second, what velocity, according to the law you have just written, would a falling body acquire in 3 seconds? If a body should fall freely for 3 seconds, and by some means have its motion changed to a horizontal direction, without friction or resistance, how far should it move horizontally in 1 second?

Experiment 30.—**Composition of Forces acting on a Moving Body.**—Lay a smooth tennis-ball on the corner of a hard, smooth, and level table-top. Strike it fairly with the palm of the hand directly down the length of the table. The hand itself must be stopped instantly after striking the ball by coming against the edge of the table. See how nearly you can make it follow the edge. After becoming expert at this, strike it from the same corner with the other hand, so that it will follow the edge at right angles to the first. Finally, placing the ball nicely on the corner of the table, strike it with both hands at exactly the same instant. What do you find to be the direction of the ball those times when you were sure both hands struck fairly "and squarely" at the same instant? Whatever the direction of its motion, was it found to be in a *straight* or a *curved* line? Do you find in this a proof of any part of the second law of motion? Does the velocity of the ball when moving under the influence of one hand alone, or both together, seem to be uniform, or does it increase or diminish? If your table is perfectly level, hard, varnished wood, or polished marble, and your ball is smooth and round, you may thus determine and write the law, stating what kind of velocity,—*i.e.*, whether uniform or otherwise—is imparted to a body by an *impulsive force.*

Experiment 31.—**Curved Motion.**—Shoot a marble or a ball across the level table of the previous experiment in such direction that it shall leave the table and move several feet horizontally before striking the floor. What is the shape of its path after leaving the table? What two forces act on the ball to change its position after it leaves the table? Are they both constant or both impulsive? Which of these is necessary to produce motion in a *curve?* Is the motion of a train of cars on a curved track an illustration in point?

Experiment 32.—**Second Law of Motion.**—Roll a marble or slide a coin with considerable force from a level table-top, allowing it 8, 10, or 20 feet of horizontal motion before it shall strike the floor. With the other hand *drop* a similar marble or coin perpendicularly from the edge of the table at the instant the moving one leaves the table-top. With a little practice in getting them away from the table-top at the same instant, the two pieces will strike the floor at the same time, *no matter how far horizontally* the flying one is projected. What effect has the horizontal impulsive force on the velocity earthward produced by gravity? Can you state how this illustrates the second law of motion?

Experiment 33.—**To Determine the First Law of the Pendulum.**—Hang two weights of say 2 and 4 ounces respectively by threads to firm supports. Nails in the top frame of the black-board are very good. Now make slipping-loops in the upper ends of the threads, and hang both on the same nail till the threads are made of exactly the same length,—say 2 or 3 feet,—or rather till the centres of the two weights are the same distance from the support. Now move one of the loops to a nail a foot from the other, and drawing both weights aside, let them go at the same instant. How do you conclude the *time* of a pendulum is influenced by its *weight?* Write a statement of the law.

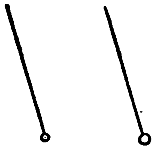

FIG. 12.

Experiment 34.—**To Determine the Second Law of the Pendulum.**—Drawing one of the pendulums aside,—say 6 or 8 inches from the perpendicular,—let it go, and count the vibrations in a half minute by the watch. Then draw the pendulum aside a much shorter distance,—say 1 or 2 inches,—let it go, and count the number of vibrations in a half minute. How do these two results compare? Try it repeatedly, and when you are sure of the relation between the *length of swing* of a given pendulum and the *time required*, write your conclusion as a law.

Experiment 35.—**To Determine the Third Law of the Pendulum.**—Carefully tie loops in both ends of three threads, and hang weights on them, so that the distances from the supports to the centre of gravity in each case respectively shall be in the ratio of 4 : 9 : 36,—say 4, 9, and 36 inches, or 12, 27, and 108 centimetres. Having hung the longest and next longest on the two nails used above, draw them aside and let them go at the same instant. Having determined the relative time of the two, hang up the very shortest, and compare its time with the longest. You thus determine the relation between the *length* of a pendulum and the time of vibration. Write it as a law.

Determine by trial the length of a pendulum to swing in 1 second. The measurement is made from the point of suspension to a point a *trifle below* the centre of gravity, the thread being very light. Having determined the time by counting for a minute or two, and having measured the length, calculate from this length, and the law last determined, the length of a pendulum to vibrate in a half second, and the length of a pendulum to vibrate in 2 seconds.

The two simple mechanical powers, lever and wheel and axle, illustrate completely the "moment" of a force,—*i.e.*, its ability to *produce rotation* about any given point or axis.

Experiment 36.—**Levers of the First Class.**—For a lever use a stick about 20 inches, or 50 centimetres, long, about 1 inch wide, and half as thick, graduated to inches or centimetres. For fulcrum, use a triangular wooden prism 1¼ inches on a side and 2 inches long. Place the lever on its side over the fulcrum at say the 10-centimetre mark. It will be necessary to have a few pieces of sheet-lead cut into strips an inch in width, and of such thickness that they may be bent up by the fingers into a casing which may readily slide along the stick. These to be used simply to balance the lever. In the case now under consideration put a piece of the lead on the short arm of the lever, and slide it backward and forward till the lever tends to stand exactly level. Then place a weight of say 100 grams right on the mark 2 centimetres from the end. This makes the short lever-arm 8 centimetres long. Balance this with a 20-gram weight on the long arm of the lever. How far do you find the 20-gram weight from the fulcrum when the lever is in equilibrium? The moment of a force being its ability to produce rotation about a fixed axis, consider the fulcrum edge as the fixed axis, and multiply each weight by its distance from the fulcrum to find its moment. How do you find the moments to compare when the lever stands level? Can you make your result agree with any law of the lever which you have ever read? In order that a system of this kind shall balance, or be in equilibrium, the moments which tend to make it rotate in one direction must just balance those which tend to make it rotate in the opposite direction. In mechanical problems those moments which tend to produce rotation *forward*, or *with* the hands of a watch, are designated by the sign +, and those which tend to cause rotation *backward* are marked —. Of course, in a case of equilibrium, as there is no rotation, the positive and negative moments are always the same.

Experiment 37.—**Bent Lever, Crank.**—Of course, a straight lever is balanced in the middle when the weights at the ends are equal. For a bent lever, take two pieces of wood of the same length, and fasten them together, as shown, by one screw, which will allow of their being set at any angle, and by being tightened hold them firmly at that angle. The fulcrum should be a pin projecting from a wall, or from a blackboard, or other board secured in a perpendicular position. Balance the bent lever in the middle. Hang a definite weight on one end,—say a pound or 2 pounds on the end *p*. Of course, this will be balanced by the same weight on the end *w*. Take the weight of *p*, and its distance from *f* in any units you please, and multiply them together for the moment of *p* when the lever is *straight*. For instance, if *p* weighs 500 grams and is 30 centimetres from *f*, its moment is 1500, and it is *measured* by the negative moment of *w*, 500 grams at 30 centimetres, minus 1500. Draw a horizontal line from *w* to *f*, even with the top edge of the lever. We wish now to find the moment of *p* when the two parts of the lever make any angle at *f*. Loosen the screw, deflect the two ends 15 or 20 degrees from a straight line, and tighten the joint. Hang a light scale-pan at each end. Place lead, nails, etc., in *p* till the arm *wf* is just level by the line previously drawn. Now replace the original weight of 500 grams in *p*, and load the scale-pan at *w* with marked weights till the arm *wf* is again level. This gives the moment of 500 grams acting at that angle. Pin a sheet of paper behind the arm *pf*, trace on it the direction *pf*; from *p* let fall a perpendicular to represent the weight and direction of the 500 grams. Measure off on this perpendicular 500 from any scale of equal parts. Resolve this into two components,—

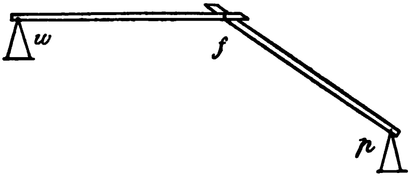

FIG. 13.

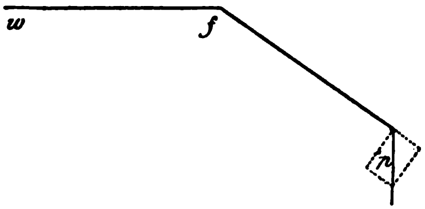

FIG. 14.

one perpendicular to *fp* and one parallel with it. Which of these is totally effective in causing rotation? How much effect has the other? Measure the effective component on the same scale of 500 parts. How does the reading compare with the number of grams in the scale-pan on the end *w*? Give the levers another angle and try again, being careful to balance the lever and scale-pans with lead, etc., in each new position till *wf* is level, before placing the weight of 500 grams in the pan at *p*. After trying other angles, see what you can do with the two arms exactly at right angles.

If you have studied geometry and plane trigonometry, calculate from a diagram drawn like Fig. 14 what the moment of a vertical force of 500 grams at a distance of 30 centimetres from the centre of rotation should be when the rotating arm is 60 degrees from the horizontal, and after getting your result see how nearly you can verify it by weights in the scale-pan at *w*.

What application of these experiments can you make to the crank of a steam-engine? In what position of the two parts of the bent lever does it illustrate the "dead point" of an engine? Draw a diagram of an ordinary steam cylinder, piston-rod, cross-head, connecting-rod, and crank, and decide whether the force of the steam on the piston may at any part of the revolution be totally effective in turning the crank. Treat an oscillating engine in the same way, and see what conclusion you arrive at.

The law of the lever, as frequently stated, is "The weight multiplied by the weight-arm is equal to the power multiplied by the power-arm." This may be kept in view in the experiments here given in the moments of forces acting on levers. It is another way of getting the result.

Experiment 38.—**Levers of Second and Third Class.**—Use here two levers, one, *ap*, balanced in the middle simply, that the whole force exerted by *p* may be directed *upward* on the levers under investigation. For second-class lever, after the apparatus is in position as shown, place a sliding lead weight on the arm near *l* till all is level. Then place a small weight on *p* and a much heavier weight at *w*, moving this weight backward and forward till equilibrium is secured. Then work out the moments of *w* and of the weight *p* considered as acting *upward* at *a*. How nearly do they agree? In similar manner verify the moments of the forces acting on the third-class lever. You will learn by practice what relative weights to use.

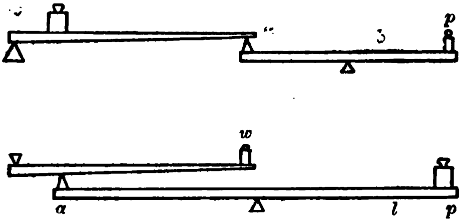

Fig. 15.

From the results of your experiments with levers of the three classes you may write the law of the respective *distances* of two given parallel forces from the point where the equilibrant must be applied. Calling the pressure on the fulcrum the equilibrant in each of the three classes of lever, determine in each case which one of the three quantities—*power*, *weight*, and *equilibrant*—is equal to the other two combined.

What is the smallest number of parallel forces, *not in the same straight line*, which will produce equilibrium?

Experiment 39.—**The Fixed Pulley.**—Hang a single pulley to a beam, or fasten it to a vertical board by a pin through it for an axle. Hang a weight of say 1 pound on each end, and determine the conditions of equilibrium with the "fixed pulley."

Experiment 40.—**The Movable Pulley.**—Arrange a second pulley on the string, as shown at *w*, Fig. 16. Balance this pulley by small pieces of lead at *p*. When the balancing is good, place a heavy weight—say 2 to 8 pounds—at *w*, and determine what weight is required at *p* to produce equilibrium. With the rope passing several times through upper and lower blocks, as shown in Fig. 17, balance the *pulley* itself by placing small weights at *p*. Then hang a very heavy weight at *w* and balance it by a weight at *p*. In each case how nearly is the ratio of the weight *w* to the power *p* expressed by the number of lengths of rope supporting *w*? Making no allowance for friction, a force of say 1 pound at *p* should be felt as a stress of how much in each of the sections of rope between *f* and *w*?

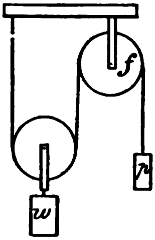

FIG. 16.

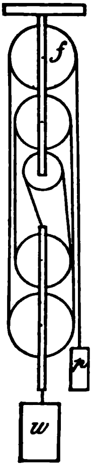

FIG. 17.

Experiment 41.—**To Determine the Law of the Inclined Plane.**—Procure a smooth, hard board, of 2 or 3 feet in length and 6 inches or more in width. Support an easy-running pulley at one end, as shown in Fig. 18. Block up one end of the board, so that there will be a definite ratio, say 4, between the length and height,—that is, *ab* : *bc* :: 4 : 1. A small car, such as is referred to in Experiment 20, is desirable, though some kind of result may be obtained with a board on rollers. Weigh the car, and always count its weight as part of the load *w*. With a weight *w* of say 4 pounds, find what weight at *p* will just balance it. The pulley at *b* should be so adjusted that the cord extends from *w* to *b*, parallel to *ab*. How does the ratio of *w* to *p* compare with the ratio of the length of the plane to its height?

Suppose your weighted car to weigh 4 pounds, and that 1 pound at *p* is found to produce equilibrium. Draw a diagram, resolving the direct upward pull or push required to sustain this weight of 4 pounds into two forces,—one of *one* pound (at *p*) and the other at right angles to the plane.

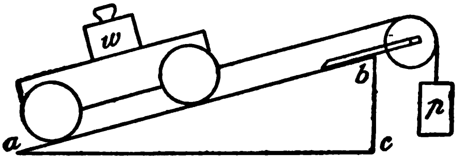

FIG. 18.

Determine the pressure of the car at right angles to the plane by calculation. This may be verified by a spring-balance, being careful to pull at right angles to the plane.

Repeat this experiment several times, varying the height of *b*.

Work out a few problems like the two following:

How much force must be exerted by an engine to sustain a load of 40 cars of freight, each weighing 30 tons, on a gradient of 1 foot in 100?

A train weighs 1,200,000 pounds. It is hauled by an engine which can exert a pressure on the train when running of 15,000 pounds. What is the steepest grade up which this engine can draw its load?

Experiment 42.—**Friction and Extent of Surface Involved.** —Procure a block the shape of a brick,—say 1 by 2 by 4 inches. Put a screw-hook in the middle of one end, attach a string, and pass it over a pulley, as in the last experiment, placing the board (the same one previously used) flat on the table-top. Hang weights at *f* until the block *w* moves

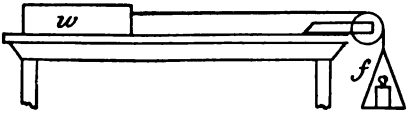

FIG. 19.

slowly and uniformly along the board. After each successive weight added at *f*, tap the block slightly to induce it to start. When you have determined the weight which will just keep *w* sliding on its flat side along the level plane, place it on its *edge*, and again determine the weight required to slide *w*, as before. Move the screw-hook, or insert another one, so that the cord from the hook to the pulley shall be parallel with the board in each case. What difference in weight required to move the block when on its edge and when on its flat side? What is the resistance to be overcome in moving the block? What increase of friction do you find on doubling the surfaces in contact, other conditions remaining the same?

Experiment 43.—**Relation of Friction to Weight.**—Use the block, board, and pulley of last experiment, only carefully weigh the block before beginning. Place on the block, as a load, a weight equal to its own weight,—*i.e.*, double the weight of the block,—and determine the weight at *f* required to keep it in motion. Continue this experiment, trebling and quadrupling the weight of *w* by loading it, and note carefully the force required to slide it. How nearly do the weight of the block and the force required to move it increase in the same ratio?

The *coefficient of friction* between two surfaces is the friction (the force required to move the body) divided by the weight of the body. Work out the coefficient of friction between smooth pine and smooth oak, or smooth pine and smooth pine, as the case may be, in each variation of these two experiments. How nearly do they agree?

Experiment 44.—**Effect of Lubricants, etc.**—Rub some good soap on the board and on the block, and again determine the coefficient of friction. Rub the soaped block on the soaped board before trying, and see that the soap surfaces do not *dry* while you are arriving at conclusions.

Try smooth brass or smooth iron on a sheet of new tinplate. Try brass on an iron plate. Oil your surfaces and try again. Devise a scheme for measuring the coefficient of friction between a skate-runner and a surface of ice. This quantity is somewhat difficult to determine, on account of the tendency of metal surfaces to freeze to ice when quite cold and under light pressure, and to melt it when warmer or under great pressure.

LIQUIDS.

Experiment 45.—**To Determine how much Pressure Liquids may Transmit.**—Procure a bottle of say 500 cubic centimetres capacity, with a neck near the bottom, and one or, better, two necks at the top. Insert a perforated rubber cork carrying a bent glass tube in the side neck and a similar cork carrying a straight glass tube in one of the top necks. Now fill the bottle, neck and all, with water. When the water has risen in the side tube to the level of the top of the bottle, insert a tight-fitting rubber cork into the other neck of the bottle, and gradually force it down, noticing the effect on the columns of water in the two tubes. This should demonstrate that liquids transmit downward pressure upward and horizontally with the same force. Without multiplying the number of necks and tubes indefinitely, we may safely infer that they transmit any pressure whatever equally in all directions. If you cannot procure the three-necked bottle, fill through the single neck at the top, carefully insert into it the cork carrying the straight tube, and exert the pressure on this cork with the fingers at the sides of the tube, when the result will be the same.

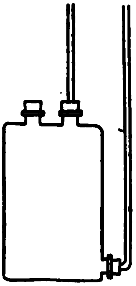

FIG. 20.

Experiment 46.—**Level of Liquids.**—Using the bottle of the last experiment, insert the cork and tube in the side neck, and partly fill the bottle with water. Notice the height of the water in the tube as compared with that in the bottle. How do the *weights* of the two compare, probably? Apply the principle determined in the last experiment, calling the small tube of water the pressure exerted. This pressure is exerted on every section of the bottle equal in area to the tube, and therefore it maintains all the sections—*i.e.*, *all the water*—at the same height. Vary this experiment by turning the bottle over and inserting a long U-shaped tube in the cork, as shown. With the water at the level of *a*, measure the distance from *a* to *b*. Take the apparatus apart, empty the tube, and pour in mercury till it fills the bottom of the tube and rises about $\frac{1}{25}$ of the distance from *b* to *a*. Without turning *ab* over, invert the bottle of water and insert the tube in the cork. This is much more easily done if you have a foot of rubber tubing from *c* to *d* and a pinch-cock to close it. As soon as the water has exerted its pressure on the mercury, measure the height of the mercury in the arm *ba*, or, more exactly, the height of the mercury *m* above the level line from *l*, where the mercury is pressed down to by the water in the other branch of the tube. Mercury is 13 or 14 times as heavy as water. How does the height of *m* compare with the distance *ab* in the first part of the experiment, the height of the water in the bottle being the same?

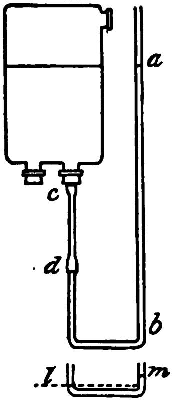

Fig. 21.

Work out such a problem as this: The pumping-station *P* (Fig. 22) forces water to a tank *T*, whose top is 100 feet perpendicularly above the pump. The engineer has a mercury gauge *m* for determining the depth of water in the tank. How *long* must the tube *m* be, or how high above the point of application of the water pressure, assuming mercury to be 13.6 times as heavy as water? *M* is graduated to read the depth of water in *T* in *feet*. How far apart must the graduation-lines be?

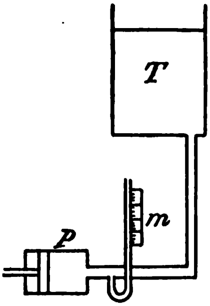

FIG. 22.

Experiment 47.—**To Determine the Amount of Water displaced by a Floating Body.**—Weigh a block of hard, heavy wood,—say hickory or oak,—about 3 inches or less in greatest diameter. Carefully place a vessel of water large enough to float the block, but not much larger, in an empty pan or basin. A fruit-can which has had the opened end melted off by being placed for a few minutes on a very hot stove-lid answers very well to float the block in. Before placing this in the basin, see that it is just level full of water. Now *carefully* place the weighed block on the surface of the water in the can. The pan or basin in which the can sits is to catch the water that runs over. Weigh this water, and *compare it with the weight of the block.* You will consider the most accurate method of weighing this water, also whether your fruit-can should be wet or dry on the outside, and whether your pan should be wet or dry when the experiment begins. Try it more than once, and average your most trustworthy results.

Experiment 48.—**To Determine the Volume of a Solid by Immersion in Water.**—Procure a block of metal of regular rectangular shape, and by careful measurement and calculation determine its volume in cubic centimetres. If you must make this block, hammer a piece of lead into the shape of a miniature brick; or, better, nail a piece of sheet-lead on the bottom of a regular wooden block, so that it will *sink* in water. Having determined the volume of this block, sink it in the can of water, as previously arranged, and weigh the overflow in *grams*. How should the number of grams of water compare with the number of cubic centimetres in the block? Suppose you should *measure* this overflowed water in a glass vessel graduated in cubic centimetres, how should this measure correspond with the volume of the block? Could you by weighing the displaced water determine the volume in cubic centimetres of any solid body, no matter how irregular, which will sink in water?

Experiment 49.—**To Determine the Volume of an Irregular Solid by Loss of Weight in Water.**—Having determined as in the last experiment the volume and weight of a regular block which will sink in water (use the same block), suspend it under your scale-pan, and weigh it while it hangs in a vessel partly full of water. The length of the string must be so adjusted that the block is entirely immersed in the water, and, of course, entirely clear of the bottom when the beam is level. This weighing may all be done with an accurate spring balance, if your laboratory does not afford a beam balance and a set of weights. Having determined the weight of the immersed body, subtract that from the original weight. How does this difference compare with the weight of the overflowed water in the last experiment? How does this difference, expressed in grams, compare with the number of cubic centimetres in the block? Can you, by weighing an irregular heavy body in air and then in water, determine the *volume* of the body?

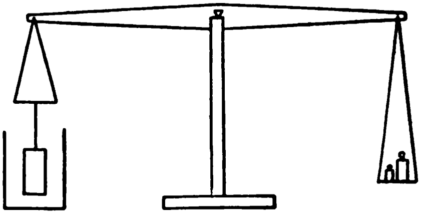

FIG. 23.

DENSITY OR SPECIFIC GRAVITY.

The word "specific," as used in physics, refers to the degree in which a given quality is possessed by a particular substance, and it is expressed by a number which compares it with the same quality in some one substance universally agreed upon as a standard. Thus, the specific gravity of a substance is the gravity or weight of that substance compared with water. The specific gravity of cast-iron is 7, which means that it is seven times as heavy as water. Similarly, the specific heat of iron is $\frac{1}{9}$, which means that it has one-ninth the capacity for heat that water has.

Experiment 50.—**To Determine the Specific Gravity of a Heavy Solid.**—The last experiment convinced us that the volume of an insoluble solid body is indicated by its loss of weight when suspended in water. Using, then, the apparatus of the last experiment, weigh a piece of stone first as it hangs in air, then as it hangs in water, and subtract the weights found. Suppose the stone to weigh 120 grams in air and 75 grams in water, the difference, 45 grams, represents not only the *volume* of the stone in cubic centimetres, but the *weight* of what? Dividing the original weight, 120 grams, by 45, we obtain $2\frac{2}{3}$ as the specific gravity of the stone. Try this with several articles till you are perfectly familiar with the operation and have a clear understanding of it. You thus determine the specific gravity of each substance tried. Compare your results with the Table of Specific Gravities given in the Appendix and see how nearly correct the table is.

Experiment 51.—**To Determine the Weight of a Regular Solid by the Specific Gravity Method.**—As a cubic centimetre of water weighs a gram, a cubic decimetre a kilogram, and a cubic metre a metric ton, it is clear that the specific gravity of any given substance expresses at once the weight of any given volume of it in the metric units of weight. Verify this with a brick. Obtain the specific gravity of a small piece, measure the brick in centimetres, and calculate its volume in cubic centimetres. The volume multiplied by the specific gravity gives the total weight of the brick. Try by weighing the brick. In obtaining specific gravity of a piece of brick, let it remain in the water a short time only, as it increases in specific gravity by absorbing water. Go to a quarry, or a bridge-pier in course of erection, and calculate the weight of the largest regular block of stone by measuring, and obtaining the specific gravity of a small piece.

Experiment 52.—**To Obtain the Specific Gravity of Insoluble Powder, Sand, etc.**—Procure a tube, graduated in cubic centimetres, or a flask with the neck so graduated. Having filled the tube or flask to a certain mark with water, carefully pour in through a long funnel a weighed quantity of the powder, and read the rise of the water in the tube. Divide this, expressed in cubic centimetres, into the original weight of the powder in grams.

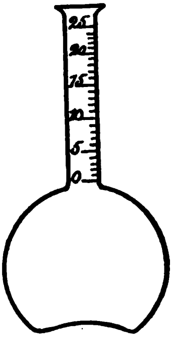

FIG. 24.

Experiment 53.—**To Find the Specific Gravity of a Liquid.**—Carefully weigh an empty flask of from 100 to 500 cubic centimetres capacity. Fill it with water to a mark made with a file on the neck, and again weigh. Pour out the water, and fill to the same mark with ether, oil, lye, syrup, or a strong solution of salt or sugar. Again weigh and compare. If you select for trial any of the articles given in the Appendix, see how nearly your results agree with the figures there given. If there is a discrepancy, it does not prove you are wrong, as liquids under the same name vary much in specific gravity. In fact, the purity or "strength" of milk, syrups, alcohol, acids, lye, ammonia, and other liquids is determined largely by testing the specific gravity.

Experiment 54.—**Second Method for Finding the Specific Gravity of Liquids.**—Weigh a suspended stone in air, in water, and in lye, brine, or syrup. The loss in one case gives the weight of a volume of water, and in the other case of the same volume of the lye, etc. Divide to obtain the specific gravity.

Experiment 55.—**Another Method.**—Procure a stick of good pine wood, about 10 inches long and ¾ of an inch square, accurately planed of the same size from end to end. Graduate this stick in millimetres, but omit figures. Oil and varnish it thoroughly, so that it will not water-soak. Tack a small piece of lead squarely on one end to induce it to float in an upright position. Having procured a tall glass jar,—a "hydrometer jar,"—partly fill it with water, and float the varnished stick in it. Observe the point to which the stick sinks, and mark it 1. Any other desired numbers for reading specific gravity may now be marked on the scale, but only within certain limits. Mark 1.25, 1.50, 1.75, and 2 *below* the unit mark, and as many as you have room for above it. The principle on which this marking is done is, that the amount of liquid displaced to float the instrument is less as the liquid is heavier, and *vice versa*. Now float your "hydrometer" in alcohol, petroleum, brine, etc., and record results, comparing them with the results obtained by both previous methods.

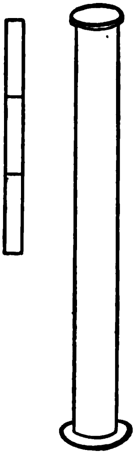

FIG. 25.

Note.—Another method of obtaining specific gravity of liquids will be given in connection with experiments on atmospheric pressure.

Experiment 56.—**To Determine the Conditions of Stability in a Boat.**—Procure a triangular prism of pine wood, about 8 inches long, with angles of say 40°, 40°, and 100°. Float this on water as a boat. With pencil, ruler, and protractor draw a triangle of the same shape and fairly large size on a sheet of ruled paper, the lines to represent the direction of the water-level. Draw it first with the top level, as in *a*, Fig. 26, then with the top inclined,—say 15°, more or less,—as in *b*. From your knowledge of geometry determine the centre of gravity of the triangle. The triangle is to be considered a cross-section through the middle of your boat, and the point determined the centre of gravity of the boat itself. Now carefully determine the water-line on the end of your boat when its top is level. Measure its position, and draw a level line across your figure to represent it. The part below this line is under water in the boat. The area of this triangle represents the water displaced, as the area of the whole triangle represents the weight of the boat. Determine the centre of gravity of this small triangle and mark it. This centre of gravity of the water displaced by a boat is called the centre of buoyancy, and changes position with every change of inclination of the boat. What is the direction between these two centres? Is the boat at rest when the two centres are thus situated? In what direction does

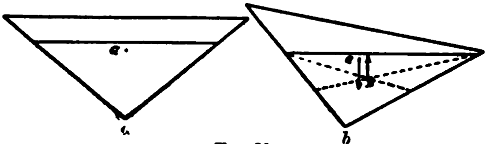

Fig. 26.

gravity act on the centre of gravity of the boat? In what direction does the water-pressure act on the boat at the centre of buoyancy? As these two forces act in the same straight line when the boat is level, can they produce any effect of rotation,—*i.e.*, any upsetting effect?

Now incline your boat 15°, and again determine the position of the water-line. (The boat may be inclined without appreciable displacement in the water by having a very light wire attached to it, and gently raising one end of the wire.) Having determined the position of the centre of buoyancy for this second position of the boat, mark it as accurately as possible on your diagram. Draw perpendicular lines through the centre of gravity and the centre of buoyancy. In which direction, up or down, do the forces act in these lines respectively? Do they constitute a couple, and if so, in which direction will they turn the boat? In practice, does or does not your boat, tilted 15°, turn as your work indicates it should,—*i.e.*, does it right itself or capsize? If an inclination of 15° fails to capsize the boat, apply a turning force till it does capsize. Observe the angle at which this occurs, draw a figure of the boat at this angle, determine the centre of buoyancy, and see which way the rotations should be by the action of this couple of forces.

Experiment 57.—**Effect of Raising Centre of Gravity.**—Take a piece of sheet-lead, wrap it around very near one end of a pine stick longer than the actual width of deck of your triangular boat, and secure by a tack. The subsequent work is easier if the *weight* of the lead is sufficient to make a safe *load* for the boat. Bore a hole in the boat, and insert the mast, lead down. Try to overturn the boat. After satisfying yourself of its stability, turn the mast upside down in the deck and try to make the boat float right side up. Find the centre of gravity of the boat as last rigged by balancing across a knife-blade. Find also the centre of buoyancy. Draw a diagram showing how the couple should act in this case at any angle you may select. If the height of the centre of gravity and the shape of the boat are so adjusted that when the boat careens the centre of buoyancy shifts more rapidly than the centre of gravity, does the boat right or capsize? What if the centre of gravity shifts more rapidly than the centre of buoyancy? Can a boat capsize if the centre of gravity is below the centre of buoyancy? Can this condition be secured by uniformity of material throughout boat and cargo, the boat being *filled* with the cargo.

Experiment 58.—**Velocity of Spouting Liquids.**—For this experiment a tin vessel, especially constructed, is recommended. Let it be cylindrical in shape, 2½ inches in diameter, and the higher the better,—say 3 to 6 feet high. Support it on a suitable base. If it is 4 feet high, have three holes made in one side, at distances of 5, 20, and 45 inches respectively from the top; or, in any event, at distances from the top bearing the ratio to each other of 1, 4, and 9. Let these be about ⅜ of an inch in diameter, and finished with necks to be corked. Procure a cork for each opening, and an extra cork fitted with a tube of about ¼ inch bore and a little longer than the cork. Fill the vessel level full of water, and, having an assistant with a pitcher of water to keep the vessel full while the experiment lasts, insert the cork with the tube in the upper neck, and catch and measure the water that runs out of this hole in a quarter of a minute, or some definite measured period of time. Record the amount, and repeat the process, trying the second opening and then the third. How do the amounts flowing from the three holes respectively in a quarter of a minute compare? What relation can you make out between these amounts and the relative distances of the

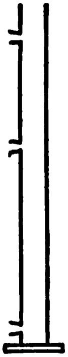
FIG. 27.

holes from the surface of the water? The object of this experiment is to obtain the relation between the velocity of a jet and its distance below the surface of a liquid. As you use the same tube, the velocities from the respective openings are exactly proportional to the quantities flowing in a given time. Can you write the result as a law?

By experiments difficult to verify with crude apparatus it has been determined that the velocity of water issuing from an orifice is the same as the velocity acquired by a body falling from the level of the surface of the water to the level of the orifice. Thus, the velocity of a jet 1 foot below the surface is 8 feet per second, of a jet 4 feet below the surface 16 feet per second, and so on, as per your law written above.

Experiment 59.—**To Calculate and Verify the Range of a Spouting Liquid.**—Set the vessel used in the last experiment on a high table or shelf. Fill it to say 4 feet above the lowest orifice. Measure the distance of this orifice above the floor, and calculate the time required for a body to fall this distance. Suppose you find this to be one-half second? The velocity of the water is say 16 feet per second. In a half second it will spout 8 feet horizontally. Place a bucket on the floor at this distance from the vertical line of your opening in the right direction, remove your thumb from the tube, and see if your stream of water strikes the bucket. By careful calculation and accurate adjustment of your tube this is easily accomplished. If the stream falls much short, the tube is so small that friction seriously interferes. In that case simply uncork the

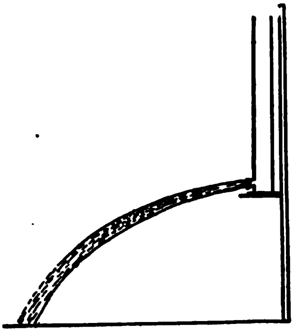

FIG. 28.

$\frac{5}{8}$-inch orifice and try it. If you are in a building supplied with water from an elevated tank by *large pipes*, this experiment may be performed by the pressure in these pipes. You may readily devise the means of securing a horizontal jet. If the elevation of your reservoir is very great, the *range* of the water will probably be less than calculated, on account of the resistance of the air at high velocities.

GASES.

Experiment 60.—**To Detect and Measure Atmospheric Pressure.**—Procure a "barometer tube" of glass, at least 33 inches long and from $\frac{1}{8}$ to $\frac{3}{16}$ of an inch in diameter. Fill this entirely full of mercury by careful pouring, close the end by firm pressure with the forefinger of one hand, and, having poured an inch of mercury into a small evaporating dish, invert the tube of mercury, with the open end under the mercury in the dish. Remove the finger from the end of the tube, meanwhile keeping the tube perpendicular. If the mercury settles in the tube, so that it stands much below 29 or 30 inches, there has probably been a leak, and you had better try it again. Have an assistant to help you measure the height of this mercury column above the level of the mercury in the dish. What is it that supports this mercury? Incline your tube to one side, and observe whether the mercury moves up or down the tube on this account. Incline your tube suddenly, and observe the effect. Right it quickly, and observe the motion of the top of the mercury. Does this illustrate a property of the mercury or a property of the air?

Experiment 61.—**To Determine the Specific Gravity of Liquids by Hare's Balanced Columns.**—(This is the method referred to at the close of Experiment 55.) You need for this experiment the apparatus shown in Fig. 29. The

"Y" tube, the long tubes (2 or 3 feet long), and the vessels are of glass. The connections are rubber tubing. To close the top rubber tube you need a pinch-cock. A common spring clothes' pin answers very well. Set under the end of one of the tubes a vessel of water, and under the other a vessel of alcohol or some other liquid to be tested. Placing the upper tube to the lips, draw air through it till the liquids have risen some distance up their respective tubes. Apply the pinch-cock, and if everything is tight, the level of the liquids will not fall. Carefully measure the heights which the liquids respectively rise above the level in the vessels. What sustains them? Which is sustained to the greater height, the lighter liquid or the heavier one? What do you thus find as the specific gravity of alcohol, brine, sulphuric acid?

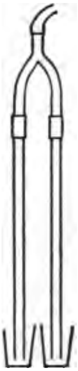
FIG. 29.

Experiment 62.—**To Determine the Specific Gravity of Air.**—Procure a metal globe made for the purpose. It should be 5 or 6 inches in diameter. Exhaust the air as thoroughly as possible, turn the stop-cock, and carefully balance the exhausted globe on a delicate beam. While it lies or hangs on the scale, open the stop-cock to allow the air to enter. Try what added weight is required to restore the level of the balance-beam. Get this very accurately. Again exhaust the globe till the weight is the same as when previously exhausted. Hold the neck under the surface of a large vessel of water, and open the stop-cock, holding the neck entirely immersed as long as water enters. Shut the cock and weigh again. You thus have the weights of equal volumes of air and water. What do you find as the specific gravity of air? What is the weight of a cubic metre of air? How many kilograms of air, and, approximately, how many pounds, are contained in your laboratory? Consult Table 3 in Appendix to see how nearly right your determination of specific gravity of air was. There is room in the experiment for a large percentage of error.

Experiment 63.—**To Prove Mariotte's Law.**—For this experiment you need a "Mariotte's tube." It is of glass, as here shown, 1 metre clear in length, with the closed arm about 20 centimetres long. Pour a little mercury into the tube. After the bend is full, so that the short arm is completely sealed, see that the mercury on both sides stands even with the lower end of the metre-rod attached to the stand as a scale. Now carefully pour in mercury. When the difference between the mercury level in the two arms is the same as your barometer reading,—say 750 millimetres,—carefully measure the length of the air column in the short arm of the tube. It is what proportional part of the original air column? If you previously found the pressure of the atmosphere to be equal to a column of mercury 750 millimetres high (or about 29.6 inches), how many times that pressure is the small confined column of air in the short arm now subjected to, with 750 millimetres of mercury compressing it, and that, in turn, pressed on by the atmosphere itself? What you find with reference to volume and pressure here, the original pressure being doubled, holds good with true gases under any pressure, and is stated in Mariotte's law, with which you are probably familiar. The principle is sometimes used in gauges for determining steam-pressure in boilers, etc. The bent tube has the open arm filled with

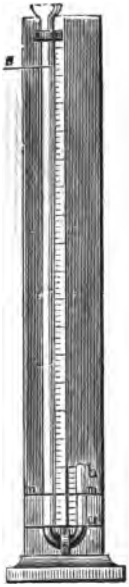

FIG. 30.

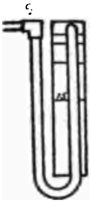

FIG. 31.

mercury, while the closed end contains air. Steam-, gas-, or water-pressure to be measured is connected at *s* (Fig. 31). Considering atmospheric pressure to be 15 pounds per square inch, and disregarding the weight of the mercury, draw a diagram of a gauge thus constructed, and graduate the scale to indicate pressures of 15, 30, 45, 60, 75, 90, and 105 pounds per square inch. Make the scale about 6 inches long.

SOUND.

Experiment 64.—**To Prove Sound a Vibration.**—Suspend a small cork ball, so that it rests lightly against one prong of a tuning-fork. Draw a violin-bow slowly and steadily across the fork, touching both prongs, till the fork gives a clear, good tone. The action of the cork convinces you that the prongs are in vibration. Change the position of the fork so that the cork hangs between the prongs while they vibrate, and notice its antics. Hold the fork by the stem in your hand, and sound it by striking it gently on your knee or a cushioned table-top. While it is sounding, convince yourself that sound is a vibration by touching it lightly against the edge of a sheet of stiff paper or other suitable object.

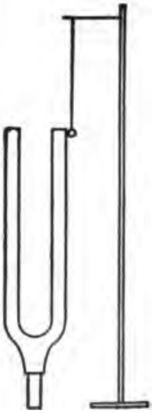

FIG. 32.

Experiment 65.—**Nodes in Vibrating Bodies.**—With the tuning-fork on its stand, use the violin-bow to set it in vibration. A touch with the finger on or near the end of the prong silences the fork. By trial, however, a point may be found near the base of the prongs where considerable pressure on both prongs fails to silence the fork. These are the nodes which are the points, or rather lines, of no vibration. The same thing may be shown with a bell. If the bell be rung by striking in a certain place, a touch with the finger in almost any point of the rim of the bell silences it. By trial, however, a point may soon be found where touching the bell with a sharp pencil-point does not silence it. After you have found this node, you can mostly rely on finding three other nodes at equal distances around the circumference of the bell. Try it. How many degrees apart are the nodes? Be sure to keep the bell going by striking in only one point while trying for the nodes.

Experiment 66.—**Nodal Lines.**—Procure a plate of hard brass, accurately cut and ground, about 10 inches square and ⅛ inch thick. At the centre, one screw should clamp it firmly to a handle the size of a trowel-handle. Taking the handle in the left hand, hold the plate level, and sprinkle well over it a teaspoonful of fine sand or iron-filings. Now draw the violin-bow carefully across the middle of one edge of the plate until it gives a clear musical note, and notice the disposition of the sand. Be sure you get it to arrange itself in lines. Do you conclude that these lines mark the *nodes* or the *vibrating segments* of the plate? Instead of sand, use a very fine powder,—lycopodium, if you can procure it, if not, a little magnesia or flour-mill dust. Where does the dust congregate when the plate vibrates? Mix the sand and dust together, and see if you can separate them by the vibration of the plate.

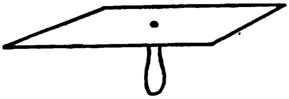

FIG. 33.

Vary the experiment by drawing the bow across the plate at or very near one corner, and notice the sand lines. This is more difficult than sounding the plate in the middle of one edge; but a few trials will give skill.

(See pages 146 and 147, S. and P., for some of the figures made by sand on square plates.)

Experiment 67.—**Alternate Phases of Vibration.**—Cut a sheet of card-board as large as your brass plate, and cutting in nearly to the middle from each corner, take out two opposite quarters, leaving only enough in the middle to hold the remaining sections in position. A little strengthening may be necessary to attach this by the centre to a light handle. Having set the plate to vibrating, quickly lay down the bow, and, taking the card-board screen just constructed in your right hand, lower it to within one or two inches of the vibrating plate. After a second, remove it to one side, and bring it back to position, repeating regularly several times. If you fail to notice a marked increase in sound while the screen is in position, persist until you do.

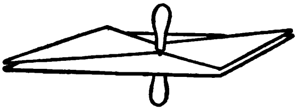

FIG. 34.

You may need a little help to properly interpret this result. Work it out yourself if you can. Consider the nodal lines as *rests*, about which the alternate sections of the plate swing, as a seesaw swings about the board it rests on. Consider also that any one of the quarters is swinging *upward*, while its neighbor on either side is swinging *downward*. You then see that the sections of the plate act as *pairs* in producing sound-waves, two opposite sections in each pair, and the two pairs always in opposite phases of vibration, tending to neutralize each other's effect. The card-board screen partly quenches the effect of one pair, and the result is now probably clear to you.

Experiment 68.—**Nature of a Sound-Wave.**—Procure about six ivory balls an inch in diameter. If these are too expensive, get glass marbles, and make the number twelve. Arrange them in a groove in a level board, as shown in Fig. 35. The cracks between floor-boards answer for the purpose. Roll

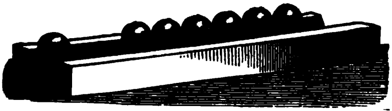

FIG. 35.

one of the balls against the end of the line. The ball at the other end should move off, and the rest retain their position. And yet the motion was communicated through the whole line. This gives us, in a rough way, an idea of the nature of a sound-wave, a slight forward and backward motion of elastic particles, each particle, by its elasticity, communicating its motion to the next, and then rebounding. The distance moved by individual air particles is infinitely greater in proportion to their size than in the case of the balls.

Experiment 69.—**To Test the Velocity of Sound in Air.**—Two persons must generally work together in this experiment. The two are to station themselves in plain view of each other, where the distance may be varied at will between limits of say 500 and 2500 feet. Each is to be provided with a watch and some means of making a loud noise. A gun is about the best. An axe and a heavy log will do. A steam-whistle is good. Use what you can command; but the observer must *see* when the sound is made. Having agreed which one is to first make the noise, let the other, at a signal, place his watch to his ear, and at the flash of the gun, or the stroke of the axe on the log, count the beats of the watch till the sound is heard. Vary the distance, taking turns in firing and timing, until you are agreed that it requires just a certain number of beats of your watch for the sound to travel the distance. The longer the distance the better. Measure it with a line, and calculate the velocity of sound in air in feet per second.

If you can command the situation, make a bargain with the engineer of a steam-mill to blow several short blasts on his whistle at intervals of ten seconds, and the whole class listen with watches.

Experiment 70.—**Sound-Waves in Solids.**—Procure a brush-handle, as long as possible,—16 feet, if you can find it. Let one student hold the box of the tuning-fork firmly against one end of the rod, while another presses the stem of the vibrating-fork against the other end. The sound comes from the box, and is carried to it by the stick. By using a ship-mast or flag-staff lying horizontally to convey sound 100 feet or more, you may easily demonstrate that wood conveys sound much more rapidly than air does. For this purpose, let one student place his ear on one end of the log, facing the other end, while another strikes the other end with a hatchet If you detect two sounds, decide which comes through the air and which through the wood.

Experiment 71.—**Velocity of Sound in Metals.**—Construct a "mechanical" telephone line of steel or copper wire 2000 feet long, with the ends in sight from each other. By having a gun discharged at one end, you may not only convince yourself that sound travels much faster through the metal than through the air, but can also get data for calculating approximately its velocity in the metal. To see how nearly correct your result is, you may, as a check experiment, run your mechanical telephone-wire through a series of elastic loops, and bend the line into horse-shoe shape, bringing the ends to such distance from each other that a sound made at one end of the line is heard at the same instant by both ears of the listener at the other end, one ear being at the telephone-receiver, the other in the air. Having previously determined the velocity in air, you have simply to measure the length of the wire in terms of the distance between its ends.

Experiment 72.—**To Determine the Number of Vibrations per Second in a Musical Note.**—Use in this experiment a rotator and siren disk, such as are furnished by some dealers in physical apparatus. The disk *d* is perforated with a certain number of holes, equidistant around its circumference;

either 48 or 96 is a convenient number. The pulley on the axle of the disk carries a projecting-pin, which strikes a thin metal spring each revolution, and gives an opportunity of counting the number of revolutions. A tube is arranged, as shown, for blowing air from the mouth through the holes in the disk. Rotate the disk and blow gently through the tube. A musical note results, rising in pitch as the speed increases. When you have reached the desired note, keep the speed uniform, and count the number of revolutions of the disk for several seconds by the watch. To be sure you have the desired note, set a tuning-fork of the required pitch to vibrating within hearing, and vary the speed of the wheel till the two sounds are alike. Instead of a tuning-fork, strike the key for middle C on a piano. When you are quite sure you have agreement of notes, and have correctly counted the vibrations, test in the same way the octave above and the octave below, and see if they come out as they should.

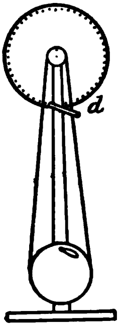

FIG. 36.

NOTE.—A "vibration" in sound signifies a motion *to and fro* of the vibrating body, as distinguished from a motion in *one direction* only, as in the case of a pendulum. In France a motion in *one* direction is called a vibration, so we divide the number stamped on a French tuning-fork by 2 to get the number of vibrations.

Experiment 73.—**Length of Sound-Waves.**—Procure a tall jar,—say 15 inches deep and 2 inches in diameter. With one hand hold a vibrating tuning-fork over the jar, and with the other pour water into it. Notice any difference in the loudness of the sound as the water increases in depth. If at a certain point there is a great outburst, pour in more water, and see what effect it has on the volume of sound. Start the fork frequently, and be sure by the *feel* that it is all the time in a state of vigorous vibration. Pour some of the water out of the jar and try again. When you have decided at what depth of water the sound is loudest (and if you make no mistake it is *several times* as loud at the right point as it is when the water is perceptibly shallower or deeper), measure the depth of the jar down to the water. If this is one-fourth the length of a sound-wave of this pitch, what is the entire length of a wave? If you have previously shown that sound travels about 1100 feet in a second, how many of these sound-waves are there in 1100 feet? Can you, from the velocity of sound in air and the number of vibrations per second in any given note, determine the length of wave of that note travelling in air? Hold the tuning-fork over or under a tube open at both ends,—say a large pasteboard tube,—and see if you can obtain an increase in sound. Take *two* tubes, each 2 feet long, and slip one into the other, telescope fashion. Determine the length for most perfect resonance, and then calculate the lengths of organ-pipe necessary for the seven octaves, calling middle C 256 vibrations.

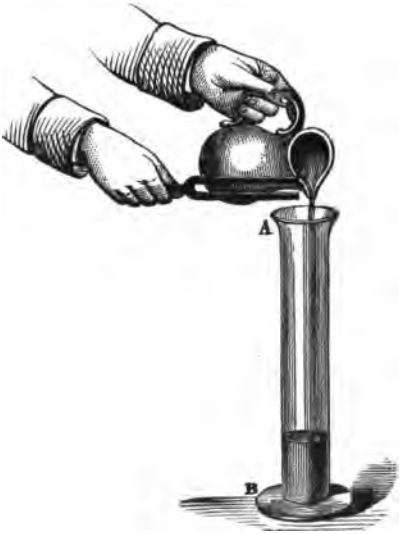

FIG. 37.

Experiment 74.—**Relation of Length of String to Number of Vibrations.**—Stretch a good violin-string between two points,—say 3 feet apart,—and excite it with a bass-bow. If your ear is not good at detecting notes, bring your rotating siren into requisition, and determine the number of vibrations per second. Then place a bridge exactly in the middle of the string, hold the string lightly to the bridge, and again determine the number of vibrations. So bridge off one-third of the length of the string, and try with the siren. The tension of the string remaining constant, do you find any relation between number of vibrations and length of string? Write it as a law.

Experiment 75.—**Relation between Tension of String and Number of Vibrations.**—Using an apparatus, such as shown in Fig. 38, stretch the cord with a force first of 1 pound, then 4, then 9. With the siren determine the number of vibrations per second made by the cord under each of the three tensions, and write a law stating how the vibrations of a given string vary with its tension.

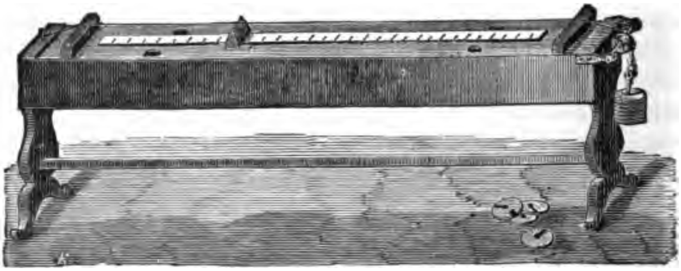

FIG. 38

LIGHT.

Experiment 76.—**Relation between Intensity of Illumination and Distance from the Source of Light.**—Cut a square piece of card-board say 6 inches on a side, and hold it midway between a naked light and a wall, so that the shadow on the wall is square. Measure accurately from the middle point, support the square card on a convenient stem, and then measure the size of the shadow on the wall. How does its area compare with the area of the card? Move the card so that it is one-third the distance from the lamp-flame to the wall, and again get the area of the shadow. On removing the card, the light which covered it a distance of say 1 foot from the lamp is immediately spread over what was the shadow on the wall,—say 2 feet from the lamp. If the area of this shadow was found to be four times the area of the card, the intensity of illumination on any part of it must have been what fraction of the intensity on the card? Similarly compare the areas illuminated at 1 foot and at 3 feet, and draw a conclusion in regard to the intensity of the illumination. This revives the law studied in your text-book about relation between intensity and distance.

Experiment 77.—**To Compare Two Lights.**—On a table very near a white wall stand on end a stick about a foot long and 1 or 2 inches in diameter a foot from the wall. Place on the table the two lights to be compared, so that they will throw sharp shadows of the stick quite close together. Move one or both of the lights back and forth until the shadows are of the same intensity of darkness. (See Fig. 142, S. and P.) Calling the lights *a* and *b*, *a's* shadow of the stick is illuminated by *b*, and *b's* shadow is illuminated by *a*, and the remainder of the wall is illuminated by both *a* and *b*, so when, by comparison, we have the shadows of the same intensity, we simply have the same illumination produced by both lights. The last experiment forced us to believe that illumination decreases with the square of the distance from the light. From that we conclude that the brightness of two lights which at different distances produce the same degree of illumination on a given surface must be to each other as the squares of their distances from that surface. Square the respective distances of your two lights from the wall, and you have the respective illuminating powers of the lights. Your result is more quickly obtained if you reduce the comparative distances to the simplest form or lowest terms and square that. Thus, suppose you found one light to be 36 inches and the other 45 inches from the wall when the shadows were pronounced equally dark, the longer distance is $\frac{5}{4}$ the shorter, therefore the farther light is $(\frac{5}{4})^2$, or $\frac{25}{16}$ times as bright as the nearer one.

Experiment 78.—**To Prove the Last Experiment.**—Procure five candles just alike. With table and stick arranged as for the last experiment, place one of the candles 2 feet from the wall, and the other four bunched 4 feet from the wall, and compare the shadows. In trying this experiment, arrange the four candles so that the light from one of the candles will have to pass *through* one or more of the flames to reach the stick, and make other bunchings, and satisfy yourself that flames are transparent to light, and that four candles give four times as much light as one.

Experiment 79.—**Plane Mirrors. Position of Image.**—It is a well-stated fact that the apparent image formed by a plane mirror is behind the mirror, and just as far behind perpendicularly as the object is in front of the mirror. To prove this in a striking manner, let two experimenters place chairs at any reasonable distance apart in a room, as at *a* and *b*. If the room is uncarpeted, the chairs should be placed on the same floor-board. Let a mirror be braced in a vertical position on a table, in the position shown at *m* in the figure, its face parallel with the floor-boards, and equally distant from *a* and *b*. Measure the perpendicular distance from *b*

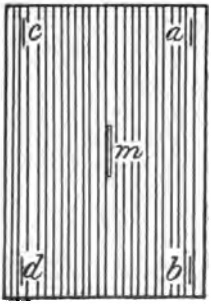

FIG. 39.

and *a* to the line over which the mirror stands. Double this distance, and place chairs *c* and *d* facing *a* and *b*. Now let the two persons sit on the chairs *a* and *b* and look towards the mirror. If the work has been accurately done, each one (and part of his chair) has been transferred to the vacant chair behind his end of the mirror. When this has been accomplished, let each one in turn leave his chair and move about within the range allowed by the size of the mirror, and see whether *his* motion can make the image of his companion move.

Experiment 80.—**Image in Mirror Reversed.**—Of the many illustrations of this fact, the following is one of the most striking. If you can find a store on a street-corner with a plate-glass window, having two plates at right angles, take a position near one of these plates on one of the streets, and look diagonally through the bulk window at the procession on the other street. You will get the reflection of objects behind you, as well as the objects themselves before you. If you will trace these reflections right up to the plane of the glass giving the reflection, you will have some very startling collisions. A trolley-car, for instance, and its image cut right through each other from end to end. In fact, every moving object and its image thus cut each other. If you are not located near a bulk-window, get a piece of good plate-glass and set it up vertically and perpendicular to a line of travel,—a railroad is the best, a trolley-line is excellent, but anything will do. You will learn how much to screen the back of your plate-glass from undesired light. It is better for the car to approach you from behind.

After you have made some good observations, draw a diagram, showing the plate which caused the reflection, and the position of image and object as they pass through each other.

Experiment 81.—**Multiple Reflections.**—Procure two pieces of good plane mirror, without frame, about 3 by 6 inches, and set them on edge on the table at right angles to each other. Place any convenient object on the table in the angle of the mirrors, and notice that the object is multiplied by 4. Bring the ends of the mirror nearer together, and at 60° you have the object repeated six times; and so you may continue till, if the mirrors are placed parallel to each other, the number of reflections is increased without limit. The 90°, 60°, or 45° arrangement is used in the construction of the kaleidoscope. For your exercise in this subject take a large sheet of paper, draw two lines at right angles, to indicate the mirrors, place a dot where the object is to go between them (or between the lines extended ever so far), place another dot at random to represent the eye of the observer, and, following the well-known principle that the angle of reflection is equal to the angle of incidence, find, by construction on this paper, the positions of the three images of the object which is to be placed on the first dot. Hint: the object and the three images occupy the corners of a *rectangle*, and one of the images is produced by reflection from *both* mirrors, as roughly indicated in Fig. 40. When the construction is completed, take four sticks like penholders, insert each in a spool for a base, stand one on the dot which indicates the object and one on each of the points found for the three images. Stand the mirrors in position, with their *backs* on the lines marked for them, and see how nearly the objects and their reflections coincide.

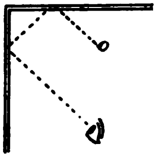
FIG. 40.

Experiment 82.—**Index of Refraction**—For experiments in refraction a specially constructed glass tank is very desirable and strongly recommended, but it is somewhat expensive, and a good round bottle of clear glass may be used instead. The bottle should be 4 or 5 inches in diameter, and no matter how shallow. Fill the bottle exactly *half full* of water, determined by its filling just half the neck when it is corked and lying on a level table. Support this horizontally on crotches, so that you may freely work under it. Draw on the bottom of your bottle two lines perpendicular to each other at the centre of the circle. When the water has come to rest in the bottle, it must just coincide with one of these lines. Admit a very narrow beam of sunlight into a darkened room, and place the horizontal bottle on the floor or table, square across the path of the beam, so that it may strike the water just opposite the intersection of these two lines. Observe where the refracted beam emerges from the under side of the bottle, also, incidentally, where the reflected part of the light leaves the top of the bottle. Mark these two points, as well as the point where the direct sunbeam enters the bottle. Suppose *a*, Fig. 42, is where the beam enters the bottle and *c* where it emerges. Measure as accurately as you can the perpendicular distances *ab* and *cd*. The reflected part of the beam, of course, makes an angle equal to *aIb*, and you measure its distance from *b* as a check on your work. *ab* is the *sine* of the angle *aIb*, or incident angle, and *cd* is the sine of *cId*, or *refracted* angle.

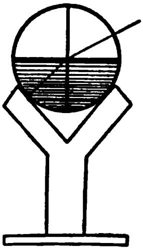

FIG. 41.

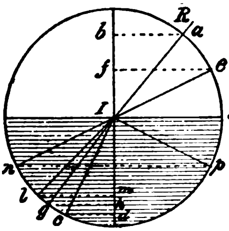

FIG. 42.

When these have been recorded, take a very good mirror of thin glass, and, supporting it firmly, catch the beam of sunlight and send it to the table at quite a different angle. It will probably be most satisfactory to catch it quite squarely and send it back towards the window.

Arrange the bottle and mirror so that the angle *alb* shall be much greater than it was before,—say equal to *elf*. Having marked the points as before, measure the sines *ef* and *gh*, and see if you can verify the "law of sines," which is, that when a beam of light passes from one of two given substances into the other, at any angle whatever, the *sines* of the incident and refracted angles maintain a constant ratio to each other. In the cases just tried, you should find *ab* to *cd* and *ef* to *gh* as 4 to 3. Having determined this, mark a spot on your bottle for the beam to enter, and measure its distance from the vertical line. Measure three-fourths of this distance for a sine on the lower part of the bottle and mark the spot. Then, with your mirror, send in a beam, so that it will properly pass through the first spot and centre, and it should emerge at the lower spot.

If for this experiment you are so fortunate as to have a graduated glass tank, you may use a printed table of natural sines (Table 7, Appendix) instead of a scale for obtaining the distances *ab*, etc.

Experiment 83.—**Reversal of Beam.**—Arrange your mirror so as to throw the sunbeam *upward* through the water, so that it will enter at *c*, *g*, etc., Fig. 42. Mark the points of emergence *a*, *e*, etc., and see if you can invert the preceding ratio. It is best for both of these experiments to have a narrow slit in dark card-board to place just where the beam enters the bottle, to cut off light which will scatter from the beam.

Experiment 84.—**To Strike an Object Under Water.**—Drop a coin into a shallow pan full of water. Tack a piece of sheet-lead to the end of a small straight stick, and stand it perpendicularly in the water to measure sines from. This must have a graduated quadrant attached to it level with the water. Place the eye in such position, some distance from the pan and slightly above the level of the water, that the coin comes into view apparently much above the true bottom of the vessel. Move your standard so that it pierces the surface just where your line of sight does. Note on your quadrant where the line of sight is. Measure the sine. Measure also a sine ¾ as long, and a straight pointer through the degree-mark of this sine, and the centre of the quadrant will strike the coin. (See Fig. 43.)

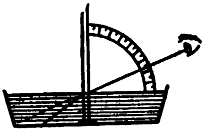

FIG. 43.

Experiment 85.—**Critical or Limiting Angle and Total Reflection.**—Using again the apparatus of Experiment 82, place the table in such position that the mirror may catch the beam of light below the level of the tank and send it *upward* through the water, as from *g* to *I*. Notice the point *e* of emergence from the bottle, and *measure* the sines *gh*, *ef*. Suppose they bear the ratio of 3 to 4. Measure off from the vertical line *Id* a sine *lm* equal to ¾ the radius *Is*, and by careful adjustment of the mirror send a beam of light through the point *l* to *I*. According to the law of sines, this beam should come out on the line *Is*, which is the surface of the water. Vary the inclination of your mirror slightly, so that the beam may pass into the water alternately above and below the point *l*, always taking the direction of the centre *I*. See how nearly the beam follows the line *lI* at the point where refraction ceases and total reflection takes place. Measure the angle of the beam with the normal (angle *lIm*) at the moment when total reflection takes place. This is the limiting or critical angle for water. After total reflection takes place, and the beam is seen emerging from the water on the other side of the normal, as at *p*, measure the angles of incidence and reflection, say *nIm* and *pIm*, and see how nearly they correspond.

In what direction must the horizon of a *fish* appear to be?

Experiment 86.—**To Find the Focal Length of a Convex Lens.**—If the sun is shining, hold the lens in such position between the sun and a plastered wall as to make the smallest definite image of the sun on the wall. Measure the distance of this image from the lens. It is the focal length.

To verify this, or to try the experiment on a cloudy day, stand between a light window and the opposite wall of the room, and so hold the lens as to make a distinct image of the window, of a tree, or something, on the wall. The distance from wall to lens when the image is "sharpest" is the focal length of the lens.

Experiment 87.—**Conjugate Focuses of Convex Lens.**—Let one light only illuminate a room. If it is day-time, let the room be darkened and a lamp or gas-jet be lighted, or one electric light turned on. Select a wall more than four times as far from the light as the focal length of the lens used in the preceding experiment. Hold the lens near the wall, so that the light from the lamp may shine squarely through it, and move it slowly towards the lamp till a perfect inverted image of the lamp-shade or flame or incandescent carbon appears on the wall. Measure the distance of the lens from the wall. Now keep on moving the lens nearer to the lamp, and see how many more positions may be found which will give a perfect image on the wall. If you find one, measure the distance of the lens, this time from the light. How does this compare with the distance of the lens from the image in the other case? The light and the wall are in the "conjugate focuses" of the lens. Move the light, if possible, some distance farther from the wall, and repeat the experiment.

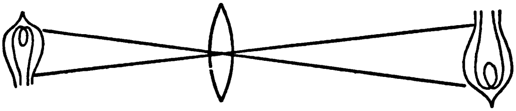

FIG. 44.

Experiment 88.—**Relative Size of Image and Object.**—With the apparatus as used in the last experiment carefully measure the size of the image and the size of the flame or lamp-shade. How do the relative sizes compare with the relative distances from the lens? Write a statement of the relative size of image and object depending on their respective distances from the lens.

Experiment 89.—**Virtual Image by Convex Lens. "Magnifying-Glass."**—Having determined the focal length of a convex lens, hold it over a printed page or other object at a distance not greater than its focal length, and, looking through it, notice the increase in the apparent size of the object. The appearance in the lens is the "apparent" or "virtual" image, because it appears only on looking through the lens. Fig. 45 shows the course of two rays from each end of the arrow to the eye. Transfer this figure to a sheet of paper, and with a pencil and ruler determine where each end of the arrow appears to be, and whether or not it appears magnified. Draw a diagram, placing the arrow beyond the principal focus of the lens, and determine whether a virtual image may be seen. Try it with the lens.

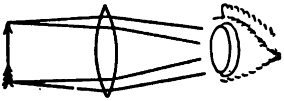

FIG. 45.

Experiment 90.—**Principle of the Telescope.**—Use two convex lenses,—a larger one of say 12- to 36-inch focus and a smaller one of say 2-inch focus. Mount the lenses in blocks made to slide on a graduated stick. Mount a screen *s*, preferably of ground glass, in a similar block within

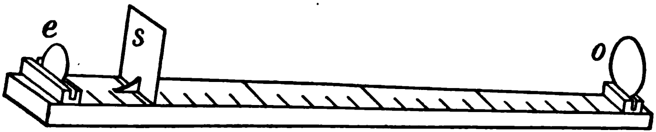

FIG. 46.

about 2 inches of one end of the graduated stick, and arrange the large lens *O* directed towards a window, so that it will form a distinct image of some bright out-door object on the screen *s*. Arrange the lens *e* at its focal length from *s*, and look through it at the inverted image on *s*. Now remove *s* and look again. With a little adjustment of *e*, a very plain image appears. This is a rude telescope. Which is the objective and which the eye-piece?

Experiment 91.—**Principle of the Microscope.**—Use the same lenses. Darken the room. Arrange the rod and lenses on a table, so that the smaller lens may be a trifle more than its focal length from a window-pane. Stick a wet wing of dragon-fly, or something thin and nearly transparent, on the window-pane in front of this lens. Shade the rest of the window pretty thoroughly. Arrange the screen *s* and the small lens *e* so that an image of the wing may appear on *s* when it is 1 or 2 feet from *e*. Examine this image by looking through the larger lens. When a good image is obtained, remove *s*, and if you have been successful in the relative positions of lenses and object, you have a rude microscope. Which is the objective and which the eye-piece? Why darken the room for this experiment and not for the preceding one?

If you properly interpreted the results of Experiment 88 you learned that a real image is larger than the object only when it is a greater distance from the lens than the object is. In which instrument, then, telescope or microscope, is the first image larger than the object? Can you think of conditions under which the telescopic image of an object would appear larger than the object itself really is? If so, make a diagram of it.

Experiment 92.—**To Measure the Magnifying Power of a Telescope.**—Take a station several hundred feet from a brick wall on which the sun is shining. Focus the telescope on a brick somewhere in the wall, and when the image of the one brick is well fixed in one eye, open the other eye and look along the outside of the tube at the wall, keeping sight the while of the telescopic image. With a little practice you may with the eye which is looking through the telescope see the image of one brick extending across 10, 20, or 50 bricks, as seen with the naked eye. Count these. A magnified brick is the distance from the bottom of one brick to the bottom of the brick next above, including one seam of mortar. This method may be employed with any spy-glass or opera-glass, in the last case using one tube only. The principle is the same with the microscope, but the application is more difficult.

Note.—Dispersion of Light and the various classes of Spectra are so fully discussed in Sharpless and Philips's "Natural Philosophy" that the subject is not taken up here.

Experiment 93.—**Primary Colors. Compound Colors. The Color Sense.**—For experiments in color, procure of a dealer in optical goods a series of colored gelatine films, representing as nearly as possible the colors of the solar spectrum. Especially try to have red, yellow, green, and blue as pure as possible. Violet is hard to obtain, but many instructive experiments may be performed by using purple gelatines, purple containing violet and red. Procure two table-lamps of good illuminating power. Set over the chimney of each lamp a short joint of stove-pipe,

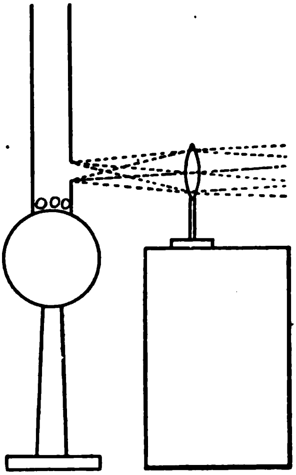

Fig. 47.

having a circle of holes around the bottom for the admission of air, and a circular opening say one inch in diameter exactly on a level with the flame of the lamp. Place a convex lens before each lamp, so that it will project an image of the upper hole on the wall. It will pay to have the lenses mounted on a stem and so placed that they may slide horizontally, as shown in Fig. 47. Have the lenses matched, if possible, and arrange the two lamps to throw their images on nearly the same spot on the wall, as shown in Fig. 48.

Now hold in front of one lens a red gelatine and in front of the other a green gelatine, so that they will color the circles of light on the wall. If, where the colors overlap, either one of the two original colors shows distinctly, lower the flame of that lamp a trifle, and when both the red and green disappear in the overlapping part a good yellow should appear in their stead.

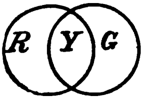

FIG. 48.

If you succeed in obtaining a violet gelatine, repeat the above experiment, using violet and green instead of red and green, and you obtain a blue in the overlapping field.

Next use your blue and yellow gelatines. You should get a good white, being careful to so adjust the relative brilliancy of the two lights that neither is in excess.

Use next a good green and a good purple gelatine, and get a white field where they overlap.

Try to produce red or green by thus overlapping any of the other colors,—that is, any colors which contain neither red nor green.

Discussion of this experiment: First, we project on the wall a distinct red disk and a distinct green disk. The eye distinguishes both where they are separate, but where the fields overlap, both the red and the green are sent to the eye at the same time, and the effect is yellow. Similarly green and violet produce blue. Neither red, green, nor violet can be produced by the combination of other colors. It is thus proved that they are primary colors. Yellow and blue and their variations, orange and indigo, are produced by combining red, green, and violet. They are, therefore, compound or rather composite colors. The three primary colors, properly combined, produce white. This is shown by overlapping green and purple, the purple containing red and violet, and by overlapping blue and yellow, the blue consisting of green and violet and the yellow of red and green.

This experiment confirms the theory that the sense of color is supplied by three classes or grades of nerve-fibres, one responding most readily to the vibrations which give the sensation of red, one to those which give the sensation of green, and one to those which give the sensation of violet, for these three, observed simultaneously and in the right proportions by the eye, give us the appearance of white, which is made by all the colors, and by omitting one, or varying the proportions of the three, all other colors are produced.

Experiment 94.—**One Color Produced by Suppressing Another. Complementary Colors.**—If you can find it, procure a Newton's color disk. On rotating it rapidly all the colors are made to blend in the eye, and a good white is the result. Now cut pieces of black muslin to cover all the segments of any one color, say green. When this is done, rotate the disk in a strong white light, and note particularly the remaining hue, which is that produced by combining all the spectrum colors, except the one quenched by the black muslin. If the whole disk when rotated gave a good white, the hue obtained by rotating with one or more of the colors suppressed is evidently the "complement" of the suppressed color. Try again some of the combinations of Experiment 93, with special reference to procuring good complements. You cannot get white without red, green, and violet somehow combined in your two gelatines. Violet, however, is so faint to the eye that most combinations of red and green are nearly white.

Experiment 95.—**Two Sensations Combined in the Brain.** —Hold a colored gelatine, say blue, over and very close to one eye, and with the other eye closed, look at a white and very bright surface at some distance. Of course, the effect is blue, the gelatine passing only such rays as produce that impression on the optic nerve and through it on the brain. Now cover the other eye with a good yellow gelatine and look through it only at the same white surface. For a similar reason the effect is yellow. Finally, cover one eye with the blue and the other with the yellow, and look through both at once at the white wall. You should get a good white, subdued, of course, because of the large amount of light absorbed by the gelatines, but with no color remaining. If either color does remain, the gelatine of that color is transmitting too much light. Try doubling that one or make another selection of colors. The experiment is bound to succeed if the conditions are right.

Discussion of this experiment: Conscious sensations, through whatever channel they come, are produced only in the brain. Experiment 93 showed that when both eyes are receiving from the same spot at the same time the vibrations which give the sensation of blue and those which give the sensation of yellow, the result is the sensation of white. Experiment 95 shows that the sensation of white is the result of the blue and yellow vibrations acting on the *brain* at the same time, for they come to the brain by different routes, the separate nerve-fibres being traceable through the point of decussation.

Experiment 96.—**Difference between Combining Colors and Mixing Pigments.**—Using the blue and yellow gelatines, which in Experiments 93 and 95 gave the best white, place one over the other, and pass light through *both*, either by looking through them at the white wall, or by placing both in front of the same lamp and projecting the image on the wall. The result should be a good green. If you cannot explain this, see Sharpless and Philips's "Natural Philosophy," p. 204.

NOTE.—For instructive elementary experiments in Polarized Light, see Sharpless and Philips's "Natural Philosophy," pp. 205–209.

HEAT.

Experiment 97.—**To Verify a Common Thermometer.**—Procure a thermometer graduated all the way from the freezing-point to the boiling-point. Put a pint of water in a vessel surrounded by an ordinary freezing mixture of ice and salt. Stir this slowly with the thermometer till it is quite full of pieces of floating ice and note the reading of the thermometer. Place the same vessel or another one where the water will boil, and when it is fairly boiling insert the thermometer very slowly, starting high up in the steam. You will immediately discover how to hold it to avoid scalding your hand with the steam. If the reading is not what it should be, examine the barometer. If that stands at about 29.9 inches, or 760 millimetres, the thermometer is probably in error. Increase of pressure raises the boiling-point of water at the rate of about 1 degree Centigrade for 27 millimetres of barometric height, or 1 degree Fahrenheit for $\frac{6}{10}$ of an inch. If your thermometer is sensibly in error, mark its error, and if you cannot procure a good one, apply the necessary correction when using the instrument, if accuracy is desired.

Experiment 98.—**To Measure the Linear Expansion of Metals**—This requires carefully adjusted apparatus and accurate work. The apparatus had better be purchased of a dealer in such goods. If you have to construct it, follow these directions in the main. *t* is a tube of tin about 2 feet long and 1 inch in diameter. It has three short tubes, set in as shown, the middle one for a thermometer, one for the admission of steam, and one for the water to drip out. Passing through the middle of this tube is the metal tube or rod to be tested. It is about ¼ inch in diameter, ½ inch longer than the tube, one end ground square and the other to a chisel edge. This passes through short perforated corks in the ends of the tin tube. The tin tube is supported on a grooved block near each end. The square end of the metal rod to be tested is pressed by a set screw *s* in a firm pillar. The chisel edge rests in a notch at *n*. This may, while in use, bear on *n* and take the weight of *t* from the block at that end. *ipn* is a light metal index pivoted at *p*, the distance *pi* being just twenty times *pn*, measured accurately from the *centre* of the pivot. Be sure the chisel edge presses on the point you measure to. The graduations on the scale *m* are in units corresponding to those your rod is measured in, millimetres or fractions of an inch. *b* is a boiler or flask connected by a rubber tube with *t*. When all is adjusted, turn the screw *s* till the index is at the zero of your scale. Note the reading of the thermometer. Pass steam more or less rapidly through the tube *t*. When the thermometer shows an elevated temperature which can be maintained for some minutes, the index also remaining stationary (be careful

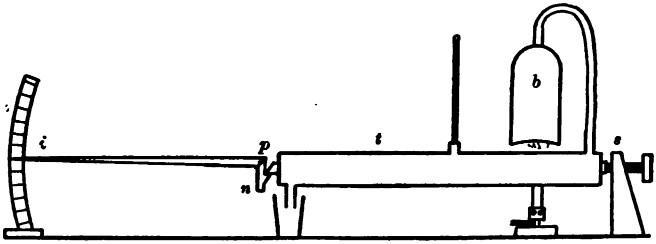

FIG. 49.

not to jostle the instrument), note the reading of the thermometer, also the position of *i*. From the rise of *i* calculate the total expansion of the rod. Write this as a decimal of its length when the experiment began. Divide this by the total rise of temperature to obtain the coefficient of expansion of the substance. If you wish your results to agree with those obtained by others, use Centigrade degrees. Try in this way brass and iron.

Experiment 99.—**To Determine the Coefficient of Expansion of Air.**—Use the tin tube and thermometer, the steam flask and connections of the last experiment, replace the brass rod which was then to be tested by a piece of thermometer tubing, and dispense with the index and graduated scale. The thermometer tube is simply closed at one end without a bulb. A bend at right angles is made within about an inch of the other end. Heat the tube with steam from the flask. After a few minutes dip the open end into a very small dish of mercury, turn off the steam, and cool the tube. Allow a small portion of mercury, say $\frac{1}{16}$ inch in length, to enter the tube as it cools, and immediately remove the dish, so that no more can enter. The piston thus formed is simply to act as a marker for the portion of air confined in the tube. Now place the tube horizontally in a trough of snow or crushed ice or freezing water. Be careful that no water gets into the open end. When the mercury comes to rest, mark the position of the inside end of the piston. Hold the thumbnail at the place while you raise the tube; then mark the place with a file. Be careful about the horizontal position of the tube when you indicate the place of the mercury. Be sure your melting ice is just at 0° C. when you mark your freezing-point as above. Try it with a standard thermometer, and add a little salt to your melting ice if necessary. Make your file-mark, however, after the mercury has stood still for an appreciable time with the thermometer indicating 0° C. The distance of this file-mark from the closed end of the tube is the unit volume of the small column of air at zero. Now heat the tube again to any desired temperature,—*e.g.*, lay it on a horizontal table, with the thermometer near it (and dry). When the mercury piston has come to rest, measure the length of the air column. The increase in length beyond the freezing-point divided by the length of the column *at* the freezing-point (determined by the file-mark) gives the total ratio of expansion, and this divided by the rise of temperature in degrees gives the fractional expansion per degree, or the coefficient of expansion of air. Is this *linear* or *cubical* expansion? Try this at several temperatures, say 20°, 50°, and 80° C., and compare your results. At what temperature should your column of air be doubled in length? At what temperature should it contract to nothing? What is the absolute zero of temperature? What is the absolute temperature of a substance or body? At what temperature does air liquefy at normal pressure? Why is it essential in the above experiment to keep the tube *horizontal* when measurements are being taken?

Experiment 100.—**To Determine the Specific Heat of a Substance. Method of Time.**—Place on a stove where a moderate heat may be preserved about uniform, first a vessel containing a kilogram of water whose temperature has been accurately determined. Stir it with a thermometer while the temperature rises say 10° C. Remove from the stove, note the exact time it has been there, but allow the thermometer to rise as high as it will by keeping it in the water a few moments longer. Each degree rise of one kilogram of water indicates one thermal unit, or *calorie.* This is a definite quantity of heat absorbed by the water, and indicates also the capacity of water for heat. Divide the total rise of temperature into the number of seconds the water was on the stove. This gives the time required for that surface to communicate one unit of heat to one kilogram of water. Now place in the same vessel one kilogram of some other *safe* liquid,—*e.g.*, cottonseed oil, lye, brine, sulphuric acid, mercury. Be sure you are regulating the stove-heat about perfectly. Determine the number of seconds required to heat the kilo of each substance one degree. The time in each case, compared with (divided by) the time required to heat the water, gives the capacity of each substance for heat, or its "specific heat." You should find each of these substances to heat more *rapidly* than water, indicating less capacity for heat, and a specific heat less than unity. Is it essential that we use exactly one kilo of each substance in this experiment? Is it essential that we use equal quantities of all?

Experiment 101.—**Specific Heat by Method of Mixtures.**—Mix equal quantities of water at different temperatures in a *very thin* metal vessel. If you cannot procure a "calorimeter" from a dealer in physical supplies, get a cup used for milk-shake. Note the temperature immediately after the mixture. Similarly try equal weights of water and oil, or water and mercury, shaking them together for a moment. In the case of the two quantities of water, how near was the resulting temperature the *average* of the two? In the case of water and oil, or water and mercury, which changed most in coming to the resulting temperature? How did the comparative changes here compare with the comparative rise of temperature in equal times as determined for the same substances by the previous experiment? For instance, suppose the water was at 15° C. and the mercury at 77°, and after mixing the temperature was found to be 17°. The mercury cooled 60°, whereas the water was heated only 2°. The mercury changed thirty times as much as the water did. In the previous experiment did it heat thirty times as rapidly? To determine the specific heat of a solid such as iron, use it preferably in small pieces, tacks, or fine nails. For this experiment you need not be particular about the purity of the iron. Weigh out a certain quantity of tacks, say 1 kilo, place them in a perforated dipper, and suspend for a few minutes in a vessel of hot water, temperature known. Take out and empty quickly into a vessel of cool water whose temperature was known. The rise in temperature of the water divided by the fall in temperature of the iron gives the specific heat of the iron.

There is much room for experimental error in experiments on heat. Radiation, conduction, the heating and cooling of containing vessels, thermometer tubes, etc., make considerable errors in final results. You may eliminate these as much as you can, and then compare your results with a table of standards. The quantities given in the tables in the Appendix are the result of painstaking work for years.

Experiment 102.—**To Determine the Latent Heat of Water.**—Place two vessels side by side on the same stove, one containing a certain quantity of water at known temperature, and the other the same weight of very finely shaved ice. Stir the ice with a thermometer till it is practically all melted; at the same time stir the water in the other vessel with another thermometer. In which is there a *steady rise* of temperature? The difference between the total rise of temperature in the two vessels is the latent heat of water, or the latent heat of fusion of ice.

Vary this experiment thus: Heat the water to the boiling-point, and pour it into the vessel containing the shaved ice. Use here the calorimeter. When the ice is melted, twice the thermometer reading subtracted from 100° gives the latent heat of fusion. If the water is not actually boiling, your calculation will be similar, but the temperature must be above 80° C.

Experiment 103.—**To Determine the Latent Heat of Evaporation of Water.**—Place a quantity of water on a stove-top of uniform temperature and note accurately the time required to bring it to a boil. The time divided by the gain in temperature of the water while coming to the boiling-point gives the time required for the water to be heated one degree. Leave the water on the same stove with uniform heat, and note the number of minutes required for the vessel to boil dry. This time divided by the time previously determined for a rise of one degree gives the number of thermal units required to vaporize one kilo of water, or, loosely, the amount of heat required to maintain water in the form of steam,—*i.e.*, the latent heat of steam.

Experiment 104.—**Another Method.**—The result of the last experiment may be arrived at thus: Fit a flask *a* with a tube, as shown. Let this flask contain any convenient quantity of water. The end of the tube passes loosely into the vessel *b*, containing say two or three times as much water as *a*, but still not full, weighed, and temperature noted. Bring the water in *a* to a boil, and maintain it so without any chance for intermission till the water in *b* is at the boiling-point or very near it. Now again weigh *b*. The increase in weight has come from *a* in the form of steam. The question is, How much heat has been carried over by the steam? Divide the increase in weight by the original weight. This decimal indicates how much steam has come over, and the increase in temperature of the water

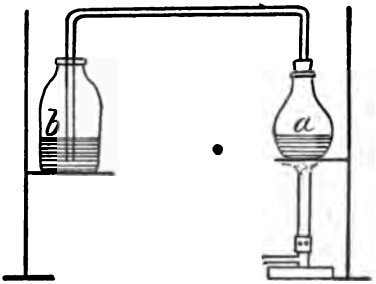

FIG. 50.

in *b*, divided by this decimal, gives the latent heat contained in that steam.

Should the steam in a radiator *condense*, or simply *pass through* at a temperature of 212° or above, to produce the maximum heating effect?

Experiment 105.—**To Determine the Dew-Point of Air.**—Fill the calorimeter, or any vessel with brightly polished nickel or silver surface, say one-third full of water at or near the temperature of the room. Test this temperature with a thermometer. If it is a warm day outside, slowly stir shaved ice into the water until the outside just begins to lose its lustre from the deposit of moisture. Note the temperature of the water. This is the dew-point, or the temperature at which the air can no longer hold invisibly the amount of moisture which it then contains, so that it begins to precipitate it. To be accurate, this experiment should be performed when the weather is *warm*, and very cold ice-water, rather than ice itself, used to pour into the warmer water in the polished vessel to reduce its temperature to the dew-point. Be careful not to pour the water in too rapidly, and be very particular not to breathe towards or near the vessel. As soon as dew *begins* to form, stop pouring in ice-water and note the temperature. Now carefully pour in a very little warm water, and if you can see the dew *begin to leave* the outside, note the temperature, and take the mean between the two for the dew-point. If this experiment is tried in winter weather, you must use salt and ice just melted, at a temperature of about 0° F., or nearly —20° C. It will be found interesting and instructive to repeat this experiment several times, with one or more days intervening each time. Try it when the out-door temperature is say 75° F., keep a record, and compare it with a determination made in a *room* of 75° when the *out-door* temperature is very low.

Suppose air at 10° F. is admitted to a room and simply heated to 75°. In what shape would moisture be deposited on your test-vessel, if at all, and why?

Experiment 106.—**Heat Absorbed by an Expanding Gas or Vapor.**—Hold a thermometer in the steam escaping from a vessel in which water is boiling without pressure,—*e.g.*, a common teakettle or a beaker-glass or an open flask. Note the temperature, and compare with the temperature of the water. If you have not a closed boiler or a model steam-engine, go to the nearest engine-house and get the engineer to open a valve in the upper part of a boiler of live steam at high pressure, so that the steam may issue in a fine jet. Hold a thermometer in the jet and note the temperature. You cannot readily try the temperature of the water in the steam boiler, but you know it is above the boiling-point of water in open vessels. Compare the temperature of the escaping and *expanding* steam with that of the steam escaping from the open vessel above and *not* expanding by its escape from the vessel. Hold your hand in the escaping jet, cautiously at first. If there is no boiler *water* escaping, the steam does not scald at all.

In which case is the steam doing *work*, when it is simply rising from a surface or when it is expanding?

If you can command a good bicycle pump and a metal globe with a stop-cock, compress air in the globe all you can. Let it lie in the room for an hour, then open the stop-cock, and let the air out against the bulb of a thermometer, and note the result. This principle repeated *in series* has recently been applied to the liquefaction of air, the air being reduced to a liquid by simply having its temperature lowered to — 191° C.

On a summer day, suppose the temperature of the air is 90° F. and the moisture deposits on a vessel of water at 75°. On account of the heated land surface, the air rises rapidly and cools to 70° by its own expansion. What should you expect to result?

ELECTRICAL CURRENTS.

Experiment 107.—**Polarization of Battery Cells.**—Construct a battery cell by immersing in a tumbler of very dilute sulphuric acid a strip of copper and a strip of amalgamated zinc. (See Exp. 177, S. and P.'s Nat. Philos.) Each of these strips should have soldered to it a piece of flexible copper wire about 1 metre in length. Connect these wires with a galvanometer, as shown in the figure. A tangent galvanometer with five or ten turns of wire on the coil is desirable. Read the deflection of the needle when the cell is fresh, and repeat the readings every few minutes. If the deflection decreases (indicating a failing of the current strength), carefully examine the surfaces of the metals and see if they are covered with bubbles of gas. With a brush or feather or by agitation clean the surface of the copper and note increase of current.

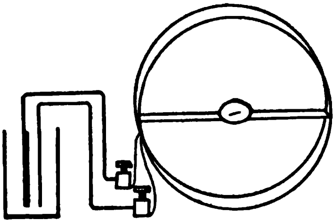
FIG. 51.

Experiment 108.—**Constancy of a Two Fluid Cell.**—Construct a "gravity" cell by placing a copper plate on the bottom of a jar and hanging a zinc plate near the top, filling the jar with water and pouring in a half pound of sulphate of copper. An insulated wire should pass from the copper plate out of the liquid. Connect this wire with the zinc plate, and after two days examine the liquid and metals carefully. Are the parts of the solution which surround the copper and the zinc respectively alike or different? Connect wires leading from the copper and the zinc to the tangent galvanometer, as in the previous experiment, and again take readings every few minutes. Are the readings essentially constant? Is the copper coated with bubbles? Does agitating the liquid increase the current? This cell may be made available without the two-days' wait by pouring a dense solution of sulphate of copper into the bottom of the vessel, and carefully pouring on top of that a solution of zinc sulphate. Try the same experiment with any other two-fluid cell,—*e.g.*, Daniell's, Bunsen's, Grenet's, etc.,—and compare the constancy with that of the single-fluid cell.

Experiment 109.—**Definite Electro-Motive Force of a Given Cell.**—Procure two Daniell's cells of different sizes, or take two of the same size, and fill the cups of one entirely and of the other about one-fourth full. Procure, if you can, a "voltmeter," or voltmeter galvanometer. If you cannot procure this, you must have, for experiments in E. M. F., or "voltage," a galvanometer of very high resistance (say 5000 ohms) and a counter force to oppose the free swing of the needle, which is generally supplied by either a spring or a strong magnet. Connect each cell separately with the galvanometer and note the readings. Are they the same? Connect the cells with the galvanometer *G*, as shown in the figure. Is the needle now deflected? See that the wires are of good size and no longer than necessary. Do not give up this experiment until you are quite satisfied that the two cells are equal in E. M. F. To compare two *different* cells—*e.g.*, a Grenet cell and a gravity cell—unite the two zincs directly, and the coppers or carbons (positive electrodes) through a galvanometer, and note which has the greater E. M. F.

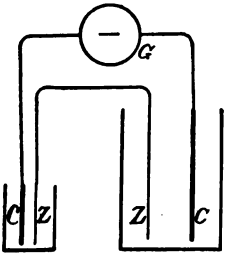

FIG. 52.

Experiment 110.—**E. M. F. Proportional to the Number of Cells in Series.**—Connect one cell of any kind with the voltmeter galvanometer, and read the deflection as closely as possible. Then connect two of the same cells in (see Art. 488, S. and P.), connect the galvanometer in cir and again read the deflection. Follow this with three, a then four, of the same cells, and verify the law that E. M. F. of a battery connected in series is directly prop tional to the number of cells.

Experiment 111.—**The Current of a Battery Proportio to its Size, or to the Number of Cells connected in "Pa lel Circuit."**—Connect one of the cells used in the las periment with an "ammeter" or ampere galvanome If this is not to be had, use a tangent galvanometer some form of galvanometer with very little resista Read the deflection of the needle. Now connect two, th four, etc., cells in parallel (S. and P., Art. 488) and read deflection each time. Have short, large wire for the c nection. Having recorded your readings, partly em the cells, so that the zincs and carbons shall be but *immersed*, and again take readings. How do they c pare with the previous readings? This is equivalen reducing the size of the cells. If you use a tangent vanometer, you must, unless the deflections are small, below 25°, multiply the deflection by the tangent of angle (see Table 7, Appendix) to get the relative defle force, or the relative amount of current. Do you prove proposition with which this experiment is introduced?

Experiment 112.—**The E. M. F. of any Number of Si Cells connected Parallel no Greater than that of One Ce** Try your voltmeter galvanometer with two, three, four. cells connected in parallel. Is there a successive rise in indicated "voltage" of the current?

Experiment 113.—**Resistance of Wires as affected Length and Weight.**—Using one cell of a battery a delicate ammeter, or tangent galvanometer, connect electrode of the cell to the galvanometer by a rather h nd short copper wire (silk-covered wire-cord is excellent), nd then connect the other electrode with the galvanomeer by a piece of fine German-silver wire, say No. 25 to 30, nd several yards in length. Wind it around a rod into a spiral"-spring shape for convenience. After noting the deflection of the needle, use successively pieces of the same wire one-half and one-fourth the length of the original piece, and note the reading of the galvanometer. Which passes the greatest amount of current? Which offers the greatest *resistance* to the current? What effect has *length* on the resistance of a conductor?

Repeat the experiment, using this time, however, equal lengths of two German-silver wires of different size, say No. 25 and No. 28. Which passes the greater amount of current and which has the greater resistance? (The resistance of the battery cell, of the copper wire, of the galvanometer, connections, etc., prevent our establishing an exact law by this method of measuring resistances.)

Experiment 114.—**To Determine Resistance by Comparison with a Known Standard.**—For this experiment use a resistance-box, such as described in Arts. 517 and 518, S. and P. If an elaborate box cannot be procured, on account of its cost, get a few coils whose resistance has been determined, and make connections with clamps, etc. Suppose you are using the regular box. and wish to determine the resistance of a given piece of German-silver wire. Connect its ends with the corners *a* and *l*, shown in the plan, Fig. 53. Connect the terminals of a dry battery cell with the points *ba*, and the wires of a galvanometer (rather delicate) with the points *gl*, unless your box is differently marked. Close the battery

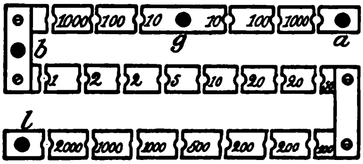

FIG. 53.

circuit key and also the galvanometer key and there will be a deflection of the galvanometer needle. Try drawing plugs from the arms *bl*, and notice whether the needle tends to come back to its original position. When it reaches its original position, draw the 10-ohm plugs from the arms *gb* and *ga*. If the needle remains at zero, and is not moved by successive closings of the circuit, the numbers of the plugs drawn from *bl*, added together, give the resistance of the wire in ohms. A study of the diagram, Fig. 54, makes this evident, the lettering corresponding with that of the plan, Fig. 53.

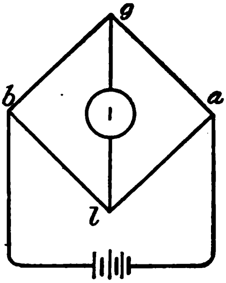

FIG. 54.

In case you cannot exactly bring the galvanometer to zero by drawing plugs from the arm *bl* when *equal* resistances have been thrown into *bg* and *ga*, you must resort to calculation, using as a basis the principle of proportional resistances, which is that the galvanometer bridge carries no current when the resistance of $bg : ga :: bl : la$. Suppose, for example, you find the resistance of your wire to be between 12 and 13 ohms. Unplug 100 ohms in *bg* and 10 ohms in *ga*. Suppose you now find that by drawing plugs in *bl* to the sum of 126 you bring the needle to zero, your unknown quantity is found to be 12.6 ohms (100 : 10 :: 126 : 12.6). Refer again to your diagram and make this perfectly clear.

Experiment 115.—**To Verify the Laws of Resistance in Conductors of Given Material.**—Carefully measure the length of the wire used in last experiment, and cut it in the middle. Try the resistance of one-half. So try other wires in whole and in half. Be sure your wires, if coiled, have the coils nowhere in contact, nor lying on a conductor.

Write a law stating the relation between total resistance and length of wire.

Now take the original length of wire and double the *size* of the wire in cross-section. If you have no means of measuring diameters, twist two wires together as a loose cable. Get the resistance of this double thick wire, and having obtained similar results with double thick wires of different materials, write a law stating the relation between resistance and cross-section of a conductor.

NOTE.—It is noticeable that, following the Brown and Sharpe (American) wire gauge, the *diameter doubles* for every difference of six of the scale, and the *area of cross-section* doubles for every difference of three. For instance, No. 10 wire has twice the diameter of No. 16 and No. 13 has twice the area in cross-section of No. 16.

Experiment 116.—**To Measure Resistance by the Method of Substitution.**—Connect one end of a long insulated fine wire whose resistance you wish to determine with one pole of a constant Daniell's cell, and the other end with a tangent galvanometer which is connected by any conductor with the other pole of the battery. When the current passes, note the deflection of the needle. Now throw out the long wire and substitute the resistance-box (the arm *bl*, Fig. 53). Unplug resistances until the deflection of the galvanometer needle is the same that it was when the wire was in circuit. The total of the resistances unplugged is the resistance of the wire sought.

Experiment 117.—**Demonstration of Ohm's Law.**—This experiment can be performed by those students only who have access to an electric-lighting system. At a suitable place in the building find a switch where you may disconnect the wires leading to a certain room for a short time, and substitute wires for your own experiment. If that is not readily accomplished, unscrew a lamp, and insert in its place a plug with wires one or two yards long. The ends of these wires give you the electric current used in the building at nearly the voltage with which it left the dynamo. If the current is not above 220 volts, you may handle these wires without danger, but on no account allow the free ends to touch *each other*.

Procure a voltmeter, if you can, connect the two wires with it, and press down the contact key for a moment, while you read the voltage of the current. If you have not a voltmeter, you may rely on the voltage marked on the lamp, or on the word of your engineer. By the method given in Experiment 114 or 116 determine the resistance of one of your electric lamps. Connect your wires with this lamp, and in series with it place an ammeter graduated to 5 amperes, and reading to tenths or hundredths of an ampere. How nearly is the number of amperes flowing through the lamp equal to the E. M. F. or "voltage" of the current, divided by the resistance of the lamp? For an ordinary lamp you will probably get a fraction of *one* ampere. For instance, with a current of 110 volts, a lamp of 220 ohms resistance passes $\frac{110}{220}$, or $\frac{1}{2}$ ampere, and gives a light of about 16 "candles."

Having obtained a definite current with *one* lamp, connect two lamps in series, and read the current. If your ammeter has graduations fine enough, connect three and four lamps in series, and read as before. Do you succeed in proving Ohm's law?

Having succeeded thus far, take your lamps out and connect two lamps in parallel,—*i.e.*, independently between the two wires,—and read the ammeter. Thus place three lamps and four. Do the reading of the ammeter and the application of Ohm's law show that the resistance is increased or diminished by adding lamps in parallel? The ammeter must be so placed (Fig. 55) that the current for all the lamps passes through it.

Are two or more lamps connected in series *lighted* by the passing current? Are they lighted when in parallel circuit? If there is a difference, can you explain it? Suppose the resistance of one lamp is 220 ohms, what is the resistance of four lamps in series and of four in parallel?

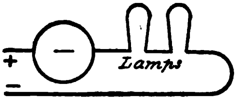

Fig. 55.

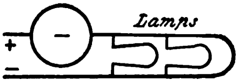

Fig. 56.

Have you ever read the term "conductance,"—the opposite or reciprocal of resistance? If *one* lamp has a certain conductance, how many times that conductance do we have in four lamps in parallel? This may help you to see why the resistance is decreased by adding lamps in parallel. It is equivalent to saying the conductance is increased.

If you are not so fortunate as to have access to an ammeter for these experiments, the principle may be established by using a tangent galvanometer, and working out the relative currents from the deflection and the tangents given in the table. (Table 7.)

APPENDIX.

TABLE I.—METRIC MEASURES.

MEASURES OF LENGTH.

		Written.	English Equivalent.
	1 millimetre	1 mm. or .001 m.	.03937 in.
10 millimetres =	1 centimetre	1 cm. or .01 m.	.3937 in.
10 centimetres =	1 decimetre	1 dm. or .1 m.	3.937 ins.
10 decimetres =	1 metre	1 m.	39.37 ins.
10 metres =	1 decametre	10 m.	32.81 ft.
10 decametres =	1 hectometre	1 hm. or 100 m.	328.1 ft.
10 hectometres =	1 kilometre	1 km.	.6214 mile.

SQUARE MEASURE.

		Written.	English Equivalent.
100 square millimetres =	1 square centimetre	cm^2.	.155 sq. in.
100 square centimetres =	1 square decimetre	dm^2.	15.5 sq. ins.
100 square decimetres =	1 square metre	m^2.	1.196 sq. yds.
100 square metres or centiares =	1 are	a.	119.6 sq. yds.
100 ares =	1 hectare	ha.	2.471 acres.

CUBIC MEASURE.

		Written.	English Equivalent.
1000 cu. millimetres =	1 cu. centimetre	c.c. or cm^3.	.061 cu. in.
1000 cu. centimetres =	1 cu. decimetre	dm^3.	61 cu. ins.
1000 cu. decimetres =	1 cu. metre	m^3.	1.308 cu. yds.

CUBIC MEASURE AS APPLIED TO LIQUIDS, GRAIN, ETC.

		Written.	English Equivalent.
10 millilitres (cu. centimetres) =	1 centilitre	cl.	.338 flu. oz.
10 centilitres =	1 decilitre	dl.	3.38 flu. ozs.
10 decilitres =	1 litre	l.	1.0567 qts.
10 litres (cu. decimetres) =	1 decalitre		2.64 gals.
10 decalitres =	1 hectolitre	hl.	2.84 bus.

CUBIC MEASURE AS APPLIED TO WOOD, STONE, AND EARTH.

Unit and only measure named, stere = 1 cu. metre = 1.308 cu. yds.

TABLE II.—METRIC WEIGHTS.

UNIT WEIGHT, DERIVED WEIGHTS, AND EQUIVALENT VOLUMES OF WATER AT TEMPERATURE OF MAXIMUM DENSITY, 4° C.

		Written.	Volume of Water.
10 milligrams	= 1 centigram	.01 g.	.01 c.c.
10 centigrams	= 1 decigram	.1 g.	.1 c.c.
10 decigrams	= 1 gram	1 g.	1 c.c.
10 grams	= 1 decagram	10 g.	10 c.c.
10 decigrams	= 1 hectogram	100 g.	100 c.c.
10 hectograms	= 1 kilogram	1 kilo.	1 dm^3. or 1 l.
10 kilograms	= 1 myriagram	10 kilos.	10 litres.
10 myriagrams	= 1 quintal	100 kilos.	100 litres.
10 quintals	= 1 tonneau	1 ton.	1 m^3.

ALSO,

		Written.	English Equivalent.
1000 milligrams	= 1 gram	g.	15.432 grains.
1000 grams	= 1 kilogram	kilo.	2.2046 lbs.
1000 kilograms	= 1 tonneau	t.	2204.6 lbs.

1 cubic centimetre or millilitre of water weighs . . . . . 1 gram.
1 cubic decimetre or litre of water weighs . . . . 1 kilogram.
1 cubic metre or kilolitre of water weighs . . . . 1 tonneau.

TABLE III.—APPROXIMATE DENSITIES OR SPECIFIC GRAVITIES.

COMMON METALS.

Aluminum	2.56	Brass	8.5
Zinc	7.2	Copper	8.8
Tin	7.3	Nickel	9.
Cast iron	7.4	Silver	10.5
Steel	7.8	Lead	11.4
Wrought iron	7.8	Gold	19.3
German silver	8.4	Platinum	21.5

MINERALS, ETC.

Charcoal	1.5	Lime	2.5
Graphite	2.25	Quartz	2.65
Diamond	3.50	Granite	2.75
Phosphorus	1.83	Glass	2.5 to 3.5
Sulphur	2.	Mica	3.
Sugar	1.61	Marble	2.72
Salt	2.28	Gutta-percha	.97
Porcelain	2.4	Ice	.92

WOODS.

Lignum-vitæ	1.34	Walnut	.65 to .7
Box	1.3	Yellow pine	.65
Mahogany	.9 to 1.05	Spruce	.5 to .65
Oak	.7 to 1.00	White pine	.55
Beech	.65 to .8	Cedar	.55
Apple	.66 to .84	Poplar	.35 to .5
Ash	.69	Cork	.24

LIQUIDS.

Ether	.74	Sea-water	1.026
Petroleum	.78 to .88	Milk	1.032
Alcohol	.81	Vinegar	1.026
Olive oil	.92	Glycerin	1.26
Turpentine	.97	Syrup	1.32

TABLE IV.—SPECIFIC HEAT. COEFFICIENTS OF EXPANSION.

	Mean Specific Heat.	Coefficient of Linear Expansion.		Specific Heat.	Coefficient of Cubical Expansion.
Gold	.032	.000015	Water	1.000	. . .
Platinum	.032	.000009	Air	.237	.00366
Tin	.054	.000023	Oxygen	.218	.00366
Silver	.056	.000019	Hydrogen	3.409	.00366
Brass	.091	.000017	Carbonic acid	.217	.00366
Zinc	.093	.000029	Mercury	.033	.00018
Copper	.093	.000017	Turpentine	.426	. . .
Iron	.113	.000012	Steam	.480	.00367
Steel	.12	.000011	Alcohol	. . .	.00111
Glass	.19	.000008	Ice	.505	. . .

TABLE V.—BROWN & SHARPE'S (AMERICAN) WIRE GAUGE.

Number.	Diameter, inches.	Diameter, mm.	Feet, per pound Copper.	Feet, per pound Iron.	Resistance, ohms per 1000 feet Copper.	Resistance, ohms per 1000 feet Iron.
1	.2893	7.348	3.95	4.45	.129	.774
2	.2576	6.554	4.99	5.61	.163	.978
3	.2294	5.827	6.29	7.08	.205	1.230
4	.2043	5.189	7.93	8.90	.259	1.554
5	.1819	4.621	10.00	11.22	.326	1 956
6	.1620	4.115	12.61	14.16	.411	2.466
7	.1443	3.665	15.90	17.80	.519	3.114
8	.1285	3.265	20.05	22.44	.654	3.924
9	.1144	2.907	25.28	28.32	.824	4.944
10	.1019	2.588	31.88	35.60	1.040	6.240
11	.0907	2.305	40.20	44.88	1.311	7.866
12	.0808	2.053	50.69	56.94	1.653	9.918
13	.0720	1.828	63.91	71.90	2.084	12.504
14	.0641	1.628	80.59	90.66	2.628	15.768
15	.0571	1.450	101.6	113.9	3.314	19.884
16	.0508	1.291	128.1	144.	4.179	25.074
17	.0453	1.152	161.6	181.3	5.269	31.614
18	.0403	1.026	203 8	227.8	6.645	41.070
19	.0359	.914	264.3	288.	8.617	51.702
20	.0320	.814	324.0	363.	10.566	63.396
21	.0285	.725	408.6	455.6	13.323	79.938
22	.0254	.645	515.1	576.	16.799	100.794
23	.0226	.576	649.7	726.	51.185	127.110
24	.0201	.513	819.2	911.2	26.713	160.278
25	.0179	.457	1033.	1152.	33.684	202.104
26	.0160	.406	1303.	1452.	42.477	254.862
27	.0142	.361	1643.	1822.	53.563	321.378
28	.0126	.321	2071.	2304.	67.542	405.252
29	.0113	.286	2612.	2904.	85.170	511.020
30	.0100	.255	3294.	3644.	107.39	644.34
31	.0089	.227	4152.	4608.	135.40	812.40
32	.0080	.202	5237.	5808.	170.76	1024.56
33	.0071	.180	6603.	7300.	215.31	1291.86
34	.0063	.160	8328.	9216.	271.58	1629.48
35	.0056	.143	10501.	11616.	342.44	2054.64
36	.0050	.127	13239.	14600.	431.71	2590.26

TABLE VI.—RELATIVE ELECTRICAL RESISTANCES OF THE METALS.

Silver	1.	Nickel	8.28
Copper	1.06	Tin	8.78
Gold	1.39	Lead	13.05
Aluminum	1.93	German silver	13.92
Zinc	3.74	Antimony	23.60
Platinum	6.02	Mercury	62.73
Iron	6.46	Bismuth	87.23

TABLE VII.—SINES AND TANGENTS.

°	Sine.	Tan.	°	Sine.	Tan.	°	Sine.	Tan.
1	.017	.017	31	.515	.601	61	.875	1.80
2	.035	.035	32	.530	.625	62	.883	1.88
3	.052	.052	33	.545	.649	63	.891	1.96
4	.070	.070	34	.559	.675	64	.899	2.05
5	.087	.087	35	.574	.700	65	.906	2.14
6	.105	.105	36	.588	.727	66	.914	2.25
7	.122	.123	37	.602	.754	67	.921	2.36
8	.139	.141	38	.616	.781	68	.927	2.48
9	.156	.158	39	.629	.810	69	.934	2.61
10	.174	.176	40	.643	.839	70	.940	2.75
11	.191	.194	41	.656	.869	71	.946	2.90
12	.208	.213	42	.669	.900	72	.951	3.08
13	.225	.231	43	.682	.933	73	.956	3.27
14	.242	.249	44	.695	.966	74	.961	3.49
15	.259	.268	45	.707	1.000	75	.966	3.73
16	.276	.287	46	.719	1.036	76	.970	4.01
17	.292	.306	47	.731	1.070	77	.974	4.33
18	.309	.325	48	.743	1.110	78	.978	4.70
19	.326	.344	49	.755	1.150	79	.982	5.14
20	.342	.364	50	.766	1.190	80	.985	5.67
21	.358	.384	51	.777	1.230	81	.988	6.31
22	.374	.404	52	.788	1.280	82	.990	7.12
23	.390	.424	53	.799	1.330	83	.993	8.14
24	.407	.445	54	.809	1.380	84	.995	9.51
25	.423	.466	55	.819	1.430	85	.996	11.43
26	.438	.488	56	.829	1.480	86	.998	14.30
27	.454	.510	57	.839	1.540	87	.999	19.08
28	.469	.532	58	.848	1.600	88	.999	28.64
29	.485	.554	59	.857	1.660	89	1.000	57.29
30	.500	.577	60	.866	1.730	90	1.000	Infinite.